Dynamic Aspects of Boiling-Heavy-Water Nuclear Reactors. Part I

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Publication date:
1966

Document Version
Publisher's PDF, also known as Version of record

Risö Report No. 126

Risö Report No. 128

Danish Atomic Energy Commission

Research Establishment Risö

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August, 1966

Sales distributors: Jul. Gjellerup, 87, Sölvgade, Copenhagen K, Denmark

Available on exchange from: Library, Danish Atomic Energy Commission, Risö, Roskilde, Denmark

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Abstract

The present work contains a general recapitulation of the elements of boiling-water-reactor dynamics. The first two chapters are devoted to a qualitative survey of the most important physical effects to be considered, and a review of the proper mathematical tools for a quantitative description. It is attempted to provide a sufficient number of literature references to cover these items in accordance with the advanced student's requirements.

In the third chapter a linearized transfer-function model of a cooling channel of a boiling-heavy-water reactor is established. A realistic example is worked through by means of a digital computer and concluded in a set of transfer-function plots.

Preface

The present report, together with Risö Report No. 129, constitutes a thesis aimed at fulfilling part of the requirements for obtaining the degree of lic. techn. in reactor physics at the Technical University of Denmark.

The studies have been carried out at the Danish Atomic Energy Commission Research Establishment Risö on a fellowship granted by the Technical University of Denmark.

My thanks are due to both institutions and are directed especially to Professor of Reactor Physics, Dr. O. Kofoed-Hansen, for his gentle inspiration and encouragement. I also want to thank P. L. Ølgaard, head of the Reactor Physics Department at Risö, who kindly allowed me to use the facilities of his department, especially the GIER digital computer. Several members of the staff of the department are gratefully borne in mind for the help they rendered through valuable discussions, especially Aksel Olsen, leader of the theoretical dynamics group.

The author

Contents

CHAPTER I

INTRODUCTION TO BOILING-WATER-REACTOR DYNAMICS

1. General Aspects of Reactor Dynamics

Reactor dynamics as an engineering discipline is becoming increasingly important in the design of nuclear reactors. This evolution emerges from the fact that the safety margin which has to be built into every new design is dependent upon the accuracy with which the transient behaviour of the plant can be predicted, especially under abnormal circumstances. Hence much investment per power unit may be saved by an advanced development of theoretical and computational methods in reactor dynamics.

The basic aspect of reactor dynamics is the neutron kinetics. Many excellent texts are available on this subject. Elementary treatments are found in any reactor textbook. For a more advanced and genuine study refs. 1 and 2 may be recommended.

For reactors operating at zero power, i. e. at a very low power, the kinetic equations provide a sufficient description of the transient behaviour of the reactor. However, when the plant produces a considerable power, several physical effects due to changes in the neutron diffusion conditions come into play. These effects are generally described in terms of several reactivity feedback paths. The pattern made up by these paths depends on the actual plant design and may be very complicated. However, it is possible to indicate theoretical and computational methods which are very general and well established. A textbook giving a good survey of this domain within reactor dynamics is ref. 3.

The engineering scientist exploring power feedback effects is invariably led to a general study of the heat-transfer and thermodynamic aspects of nuclear power plants. An excellent elementary textbook in this field is ref. 4. In many cases it will also be necessary to delve into specific texts on heat transfer, thermodynamics and two-phase fluid flow. A very comprehensive and up-to-date bibliography on these items is given in ref. 5. A classical reference on heat transfer is ref. 6.

Besides physical understanding of the dynamic behaviour of reactors, any accurate description of an actual system requires the use of advanced mathematical and numerical methods. Literature on specific methods for reactor-dynamics calculations is not directly available in textbooks, but is spread over a large number of scientific and technical reports, many of them

referring to large computer codes. References to some of these reports are made in the following text and in ref. 7. Some general references are given in chapter II.

A special aspect of reactor dynamics is the prediction of operating domains within which a specific reactor type is stable, conditionally or unconditionally. We shall return to this question in chapter II.

2. Typical Features of Boiling-Water Reactors

The main feature characterizing the dynamic response of boiling-water reactors is the formation of steam in the reactor core.

Normally the coolant, i. e. light or heavy water, is conducted through the core in vertical channels containing the fuel subassemblies. These may consist of parallel plates or clusters of rods. The plates or rods are made of fuel material and are usually clad with some metallic alloy which is resistant to radiation damage and represents an acceptable compromise as to low neutron absorption and good thermal conduction properties. The purpose of the cladding is partly to support the fuel structure mechanically and partly to prevent fission products from mixing with the coolant.

As the coolant passes from the bottom of the reactor upwards along the fuel assembly, the heat transferred from the fuel increases the temperature of the coolant and at a certain point, the boiling boundary, causes it to boil. From this point upwards the coolant will flow as a mixture of a liquid and a vapour phase. The vapour, which is generally of very low density as compared with the liquid, has a very strong feedback effect on the power performance of the reactor.

In the boiling region of the cooling channel the vapour acts on the liquid phase in such a manner as to supersede the liquid. This results in a considerable acceleration of the liquid compared with the inlet velocity in order to maintain a constant mass flow. The vapour phase itself is apt to travel faster than the liquid by a factor called the slip ratio.

In dynamic investigation it is natural to distinguish between two types of boiling-water reactor plants according to the method of circulating the coolant: forced circulation and natural circulation.

In the forced circulation it is assumed that the coolant inlet velocity or mass flow is controlled directly by means of pumps etc. The problem of dynamic performance is in this case simplified by the fact that the mass flow through the channel can be considered an independent variable.

However, as pumps are expensive, the alternative method of natural circulation is often preferred if possible. In this case the driving force is supplied by gravity, use being made of the fact that the coolant density in the recirculation loop is on an average greater than that in the reactor core. The natural driving force may be increased by introducing a chimney, i. e. an elongation of the space immediately above the upper ends of the coolant channels (see e. g. ref. 4). Thus in a natural-circulation reactor there is competition between the gravitational force and the pressure losses in the system due to friction and acceleration, resulting in the net driving force. It is easily conceivable from the above that natural-circulation dynamics, operating with coolant mass flow as a dependent variable, is much more involved than forced-circulation dynamics.

3. Core Dynamics Related to Plant Dynamics

From the above it is quite obvious that the reactor core is just one element of the large physical system subjected to investigation by the reactor dynamics engineer. Further, it is very important to emphasize that the description of the individual parts of the system should be carried out with the same degree of accuracy. Hence it is necessary to judge beforehand which details of a single part of the system will play a significant role when coupled to the rest of the system and which will not. This, however, requires a very broad experience, and in many cases where such experience is not present it must be gained by trial and error.

In the present work it is attempted to describe the dynamic properties of a cooling channel of a boiling-heavy-water reactor core. As a consequence of the above considerations it involves an attempt to select the most important physical effects and to describe these with necessary and sufficient accuracy.

In the preceding section the distinction between natural and forced circulation was pointed out. Throughout this work the coolant inlet velocity is considered an independent variable, thus restricting the use of the results to forced-circulation systems. To make the results applicable to natural-circulation systems it will be necessary to include a description of the pressure losses along the channel.

As the core is only one system out of several making up a nuclear power plant, it is not justifiable to identify its dynamic performance with the allover dynamic performance of the plant. However, it is possible to gain much useful information from core considerations alone, provided the limitations are constantly borne in mind.

4. Survey of Physical Effects Determining the Dynamic Response of the Boiling-Water-Reactor Core

(a) Neutron kinetics

The neutron kinetics are obviously the basic element in any nuclear reactor design. The question to be answered in each case is to what accuracy the description should be carried out. As is more thoroughly discussed in ref. 7, it is generally sufficient to use a one-delayed-group approximation to the set of kinetic equations.

(b) Neutron diffusion

The spatial distribution of the neutron flux in the reactor depends on the spatial distribution of the neutron diffusion parameters. As long as these parameters are constant (or vary in the same way) throughout the reactor, the thermal flux will vary as a whole (i. e. only the level, not the shape will vary). Hence the flux distribution will have no direct influence on the dynamic performance. In this case the reactor may be treated as a so-called point model. On the other hand, when the diffusion parameters vary in a highly space-dependent manner, the diffusion and slowing-down of the neutrons call for a more or less accurate description. In such cases a one- or two-energy-group model will often be preferred.

(c) Doppler effect

In reactors fuelled with natural or slightly enriched uranium, the great resonance absorption in U^{238} plays a considerable role in determining the resonance escape probability, which is a function of the resonance integral. As the fuel temperature in such reactors is raised, the resonance integral increases through the so-called Doppler effect. This gives rise to a fuel-temperature negative reactivity feedback in addition to the ordinary fuel-temperature feedback through thermal absorption as mentioned in subsection (f) below. An elementary explanation of the Doppler effect may be found e. g. in ref. 8.

(d) Heat transfer to coolant and moderator

Heat is generated in the interior of the fuel material at a rate proportional to the thermal flux. For a vertical fuel element it is most common to neglect the variation over the fuel cross section and make the axial variation proportional to the axial flux variation.

The heat generated gives rise to a temperature elevation in the fuel and to a heat flux through the fuel surface, proportional to the temperature

difference between fuel and cladding surfaces. The heat flux passes through the cladding, which usually (except for very fast transients) has a negligible heat capacity, and is absorbed by the coolant flowing along the outer surface of the cladding. The heat transfer to the coolant is in general not simply proportional to the temperature difference between cladding and coolant, see for instance refs. 4 and 9. Simple proportionality, however, may often be used as a good approximation.

A thorough analytical treatment of the heat transfer from fuel to coolant for the case of a cylindrical fuel rod can be found in ref. 7. Here it suffices to characterize the physical behaviour of the heat transmission by a delaying or inertial effect by which slow variations in heat generation are transformed to heat-flux variations in a quasistationary way, while rapid variations are subjected to considerable damping and phase shift. The time constant of this action is from 1 to 10 seconds, depending on the type of fuel material, being smaller for metallic and larger for cheramic fuels.

(e) Steam formation

As previously explained, steam is generated above the boiling boundary in the channel. In dynamic considerations it is important to note that several factors determine the position of the boiling boundary. The first of these is the amount of subcooling of the coolant, i. e. the difference between the saturation temperature (determined by the system pressure) and the coolant inlet temperature (which in this treatment is considered an independent variable). The second factor is the heat flux from fuel to coolant, which accounts for the temperature rise per unit length together with the third factor, the coolant inlet velocity or mass flow (in this treatment an independent variable). The rate of heat generation above the boiling boundary is determined by the heat flux from fuel to coolant.

The spatial distribution of the fractional steam volume above the boiling boundary is determined by the coolant velocity. The previously mentioned liquid supersession causing an acceleration of the two-phase flow has a significant influence in this connection.

It can be seen that the distribution of the transient steam volume, or steam void, over the whole cooling channel is a rather complicated function, and many more or less accurate computational methods have been developed for its determination.

Actually this function is the main characteristic of boiling-water reactor core dynamics and responsible for the fact that the point-model description (see subsection (b)) has to be abandoned as inadequate for many purposes.

(f) Temperature- and void-reactivity feedback

When the temperatures of coolant, moderator and fuel are raised, the thermal-neutron spectra in the appropriate reactor regions are shifted towards higher energies. This has a direct influence on the thermal absorption and scattering cross sections for the reactor materials, thus giving rise to a negative temperature-reactivity feedback.

In the same way density variations provide negative feedback paths. Usually density variations in solids can be neglected in this connection. The temperature variations in the moderator will usually be small enough to make the corresponding density variations small as compared with those in the coolant, because of steam formation. Hence the most important density feedback path is that of the steam void.

This effect, however, is more complicated than the temperature feedback effects because steam bubbles formed in different parts of the cooling channel may have a greatly varying influence on the reactivity. Actually the concept of reactivity is hardly applicable in this connection. Instead it is more correct to speak of local variations in the diffusion parameters. Only in special cases where the local void effects can be lumped into an overall reactivity effect by means of some weighting function, can the point-model description mentioned earlier be justified.

(g) Control system

In any reactor design there must be some means of controlling the chain reaction. Hence in a dynamic investigation it is necessary to be able somehow to simulate such a control system. However, it is desirable to make as few assumptions as possible as to the actual design of the system.

In a point-model treatment no assumptions at all are necessary since it is possible to prescribe an externally controlled reactivity function.

In a distributed model (see e. g. ref. 7) the most simple assumption to be made is that the control system (whatever design is used) acts as a distributed control absorption cross section, variable with time.

(h) Steam load

A very important factor in the dynamic performance of a boiling-water reactor is the steam-load variations.

In a core-dynamics investigation the steam load of course cannot be directly incorporated. As, however, the main effect of steam-load variations on the core is a variation in the system pressure, a usable simulation of load variations can be obtained by changing the system pressure to an

independent variable.

(i) Heavy water contra light water

What has been said above about the important physical effects in boiling-water reactors applies equally well to heavy-water and light-water reactors.

Nevertheless, the two reactor types behave quite differently in a dynamical sense.

This is due mainly to the different diffusion properties of light and heavy water. Light water has a much smaller diffusion length for thermal neutrons than heavy water.

Because of this smaller diffusion length the configuration of light-water lattices is generally more tight, which means that the relative water volume acting as coolant (i. e. subjected to steam-void formation) is larger for light-water than for heavy-water lattices.

Thus, the importance of the steam voids for the creation of negative reactivity is much greater in a light-water than in a heavy-water reactor.

Further, the axial flux shape in a light-water reactor (for the same reasons as above) exhibits a much greater tendency to oscillate about the shape corresponding to steady-state operation than that in a heavy-water reactor.

To conclude the above, the difference in core dynamics between boiling-heavy-water and boiling-light-water reactors is more quantitative than qualitative, but great enough to make quite different demands on the analytical and numerical methods to be chosen in the two cases.

CHAPTER II

METHODS OF THEORETICAL TREATMENT

1. Linear Methods. Transfer Functions

In chapter I we have outlined the main physical effects which determine the dynamic behaviour of the boiling-water-reactor core. The mathematical formulation of the theory in general involves the appearance of non-linear, partial, second-order differential equations in the independent variables representing space and time.

The exact solution of the total set of equations is usually beyond the reach of the analytical tools available. Hence, the first step to take is to eliminate a suitable number of inconvenient features, for instance the non-linearity, some or all space co-ordinates, etc.

If all the space co-ordinates are eliminated by some sort of integration over the core volume with or without the use of weighting functions, we have a point-model representation of the dynamic situation.

If the non-linear properties are removed by subtracting the steady-state values of all the physical parameters and neglecting all products of their variations, we have a linearized model.

No doubt, the most popular and best established method of treating a linearized model is the method of Laplace transformation. It is most obviously valuable when applied to linearized point models, but is also used to some extent in cases of linearized distributed models.

The characteristic feature of this method is the replacement of the time variable by a frequency parameter obtained by a transformation of the physical quantities which involves an integration along the entire positive time axis.

Many excellent texts on Laplace transformation are available. Here we shall just recommend ref. 10.

The use of the method requires tables of Laplace transforms of a large number of time functions. Ref. 10 contains a very useful table of this kind.

The method of Laplace transformation is used in chapter III to establish a linearized point model of a cooling channel of a boiling-heavy-water reactor.

By the same method, the ratios of the Laplace transforms of the dependent variables to those of the independent variables are established. Any one of these ratios is named a transfer function. They turn out as complex numbers expressing amplitude and phase response of the dependent variable when a sine wave of a given frequency is superimposed on the independent variable.

2. Semi- and Non-linear Methods. Describing Functions. Digital Computers

The method of linearization implies that only small variations of the physical parameters are allowed. Hence it is generally a very valuable tool for investigations concerning the normal operating characteristics of the reactor plant.

As, however, the aim of reactor dynamics is to explore transient responses and stability characteristics, also under extreme or accidental conditions, reliable semi- and non-linear computational methods are called for.

A general way of approach to this domain cannot be shown. A characteristic feature of a large-scale dynamical investigation is a sequence of choices of degree of accuracy, type of description, use of analytical or numerical approach, and so on. To get through this with an honest feeling that the final result is usable requires a broad background in mathematical and numerical analysis. Some general literature on the subject is listed as refs. 11 to 14. Specific literature on the analytical treatment of differential equations can be found in refs. 15 and 16. Refs. 17 to 20 may be useful for a numerical attack on systems of differential equations.

One method representing a semi-linear attack is the describing-function method. The describing-function concept is an extension of the transfer-function concept. Hence a describing function is defined as the ratio between the Laplace transforms of two physical parameters, i.e. it is assumed that a sinusoidal variation in an independent variable produces a sinusoidal variation of the same frequency in the dependent variable. However, in contrast to the transfer function, the describing function is assumed to depend on the amplitude of the independent variable perturbation. More about this function can be found in refs. 1 and 21. Generally, the describing function is a very inconvenient tool which offers little advantage over the transfer-function concept.

A method which tends more and more to become the method for treatment of great reactor-dynamics problems in a non-linear way is that of constructing a digital computer model to simulate the reactor system.

The accuracy to be obtained in this way is normally limited by two factors: computer-memory size and total computing time.

By this method the system of non-linear differential equations is transformed to a system of finite difference equations, and iteration techniques are widely utilized to overcome non-linearity and interaction problems.

Ref. 7 presents an attempt to simulate the dynamic behaviour of a cooling channel of a boiling-heavy-water reactor by means of digital computing technique. Thus, this attempt is the non-linear equivalent to the transfer-function model established in chapter III.

3. Stability Theory

As previously explained, a special discipline within reactor dynamics is the prediction of stability by analytical methods.

By stability is understood, loosely, the degree to which a reactor system is liable to return to its original state of operation after some disturbance.

The aim of stability theory is to determine intervals of reactor parameters (such as temperature-reactivity coefficients, void-reactivity coefficients, etc.) within which the reactor system is or may be stable under certain subconditions.

The theoretical work underlying this discipline is that of Lyapunoff. His efforts were directed towards a quite general theory of stability of motion and started with his doctoral dissertation published in 1892.

Lyapunoff and his co-workers recognized a number of general criteria for both stability and instability. These criteria have been used in a somewhat modified form by the reactor analysts. Since, however, the use of the Lyapunoff theory always implies very crude simplifications of the reactor systems, there seems to be a serious limit to the reliability of numerical results obtained on this basis.

On the other hand, the study of stability from this point of view provides much insight into the nature of the dynamic behaviour, unlike for instance the digital-computer approach.

A review of the works of Lyapunoff can be found in ref. 22, while some applications of the theory to reactor stability problems are reported in ref. 21.

CHAPTER III

LINEAR DYNAMIC MODEL OF A COOLING CHANNEL OF A BOILING-HEAVY-WATER REACTOR

In this chapter an attempt is made to establish a linear dynamic model of the cooling channel on the basis of the transfer-function concept as explained in chapter II, section 1.

This model is used in ref. 7 to provide a partial check on the non-linear digital-computer code BRENDA.

In its main features, the derivation, as presented below, has been taken over from ref. 23.

1. Outline of the Linear Model

Fig. 1 shows in a diagram the interaction of the most important physical quantities characterizing the dynamic behaviour of the channel.

The system appears to have 20 Laplace-transformed parameters, to which names are assigned. These names are explained in table 1. The first five parameters of the table are free parameters, i. e. their values

are not determined by the system itself, but must be specified from outside. The values of the remaining parameters are bound and can be determined from a system of linear equations in the 20 parameters which is immediately derived from fig. 1.

The solution of this system can be obtained by means of an analogue or a digital computer. In section 3 an actual example is worked out and prepared for solution on the digital computer GIER.

2. Derivation of the Partial Transfer Functions

Each box in fig. 1 represents and defines a transfer function, i.e. a ratio between two Laplace-transformed quantities.

There appear to be 27 transfer functions in the system, and these will be calculated in the following.

(a) Transfer functions A_q, A_b, A_v, A_t, V_a.

These transfer functions determine the variation of the average void in the channel with the variations of heat flux to the boiling region, boiling boundary, coolant inlet velocity and saturation temperature.

The differential equation for the void formation is derived in ref. 7, chapter I, section 4:

$$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x}(\alpha v_s) = - \frac{(1-\alpha)\rho_w c}{r \rho_s}\left(\frac{\partial T}{\partial t} + v_w \frac{\partial T}{\partial x}\right) + \frac{q}{r \rho_s} \quad , \tag{1}$$

where α, T, v_s, v_w, and q are functions of space and time.

Assuming for the moment that the first term on the right-hand side is very small compared with the second, we may reduce the equation to

$$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x}(\alpha v_s) = \frac{q}{r \rho_s} \quad . \tag{2}$$

When the variations are introduced into eq. (2), we get

$$\frac{1}{v_s(x,o)} \frac{\partial}{\partial t}\left\{ v_s(x,o)\delta\alpha(x,t)\right\} + \frac{\partial}{\partial x}\left\{v_s(x,o)\delta\alpha(x,t) + \alpha(x,o)\delta v_s(x,t)\right\}$$

$$= \frac{\delta q(x,t)}{r \rho_s} \tag{3}$$

Assuming q and v_s to be separable in space and time and putting $q(x,o) = q_o \sin(\frac{x}{L}\pi)$, we have

$$\delta v_s(x, t) = v_s(x, o) \frac{\delta v_{inlet}}{v_{inlet}} , \qquad \delta q(x, t) = q(x, o) \frac{\delta q_o}{q_o} , \tag{4}$$

and hence

$$\frac{1}{v_s(x, o)} \frac{\partial}{\partial t} \{ v_s(x, o) \delta a(x, t) \} + \frac{\partial}{\partial x} \{ v_s(x, o) \delta a(x, t) \} =$$

$$\frac{q(x, o)}{r \varrho_s} \left\{ \frac{\delta q_o}{q_o} - \frac{\delta v_{inlet}}{v_{inlet}} \right\} . \tag{5}$$

The assumption of the above sinusoidal variation of the heat flux implies that L must be the total height of the reactor core including the reflector savings H1 and Hn at the lower and the upper end respectively. Introducing the length BB as the distance from the channel inlet to the boiling zone corresponds to assigning the co-ordinate H1 + BB to the boiling boundary.

Laplace transformation of (5) and integration yield

$$v_s(x, o) \overline{\delta a}(x, s) = e^{-s \int_{H1+BB}^{x} \frac{dx''}{v_s(x'', o)}} \left\{ \int_{H1+BB}^{x} \frac{q(x', o)}{r \varrho_s} e^{s \int_{H1+BB}^{x'} \frac{dx''}{v_s(x'', o}} \right.$$

$$\left. \left\{ \frac{\overline{\delta q_o}(s)}{q_o} - \frac{\overline{\delta v_{inlet}}(s)}{v_{inlet}} \right\} \right. . \tag{6}$$

From eq. (2) we have

$$a(x, o) v_s(x, o) = \int_{H1+BB}^{x} \frac{q_m}{r \varrho_s} = \frac{q_m}{r \varrho_s} (x - H1 - BB) . \tag{7}$$

Here q_m, the average heat flux to the boiling region, has been introduced to avoid difficulties with the integration of eq. (6).

From a simple flow consideration the following relationship between steam velocity and void fraction emerges:

$$v_s(x) = \frac{sl \cdot v_{inlet}}{1 - a(x)(1 - sl \frac{\varrho s}{\varrho w})} , \tag{8}$$

where a constant slip ratio sl is assumed.

Solving (7) and (8) for $v_s(x, o)$, we have

$$v_s = sl \cdot v_{inlet} + \frac{q_m}{r\rho_s}\left(1-sl\frac{\rho_s}{\rho_w}\right)(x-H1-BB) . \qquad (9)$$

Introducing this result into (6) and integrating, we obtain

$$\overline{\delta a}(x, s) = \frac{q_m}{r\rho_s}\,\frac{1}{s+\left(1-sl\frac{\rho_s}{\rho_w}\right)\frac{q_m}{r\rho_s}}$$

$$\left\{1-\left(\frac{sl\cdot v_{inlet}}{sl\cdot v_{inlet}+\left(1-sl\frac{\rho_s}{\rho_w}\right)\frac{q_m}{r\rho_s}(x-H1-BB)}\right)^{1+\frac{s\cdot r\rho_s}{q_m\left(1-sl\frac{\rho_s}{\rho_w}\right)}}\right\}$$

$$\left\{\frac{\overline{\delta q_o}(s)}{q_o} - \frac{\overline{\delta v_{inlet}}(s)}{v_{inlet}}\right\} . \qquad (10)$$

To obtain the average void variation over the whole channel, $\overline{\delta a}(s)$, we integrate eq. (10) over the boiling region and divide it by the length of the channel, H = L-H1-Hn :

$$\overline{\delta a}(s) = \frac{\frac{HB}{H}}{1-sl\frac{\rho_s}{\rho_w}}\cdot\frac{1}{1+s\frac{r\rho_s}{q_m\left(1-sl\frac{\rho_s}{\rho_w}\right)}}$$

$$\left\{1+\frac{sl\cdot v_{inlet}}{HB}\,\frac{1}{s}\left(e^{\frac{s\cdot r\rho_s}{\left(1-sl\frac{\rho_s}{\rho_w}\right)q_m}\ln\frac{1}{1+\left(1-sl\frac{\rho_s}{\rho_w}\right)\frac{q_m HB}{r\rho_s\cdot sl\cdot v_{inlet}}}}-1\right)\right\}$$

$$\left\{\frac{\overline{\delta q_o}(s)}{q_o} - \frac{\overline{\delta v_{inlet}}(s)}{v_{inlet}}\right\} . \qquad (11)$$

Here HB = L - H1 - Hn - BB, the length of the boiling zone, has been introduced.

We have assumed the steam velocity to be separable in space and time, i. e.

$$\frac{\delta v_s(x, t)}{v_s(x, o)} = \frac{\delta v_{inlet}(t)}{v_{inlet}} \quad .$$

This, however, is not consistent with the flow consideration expressed by eq. (8), which yields on differentiation:

$$\frac{\delta v_s(x, t)}{v_s(x, o)} = \frac{\delta v_{inlet}(t)}{v_{inlet}} + \frac{1 - sl\frac{\rho_s}{\rho_w}}{1-\alpha(x, o)(1-sl\frac{\rho_s}{\rho_w})} \delta\alpha(x, t) \quad .$$

This observation necessitates the establishment of a feedback path from the void to the inlet velocity:

$$\overline{\delta v'_{inlet}}(s) = \overline{\delta v_{inlet}}(s) + V_a \overline{\delta\alpha}(s) \quad .$$

The transfer function V_a can be approximately represented by

$$V_a \sim \frac{v_{inlet}(1-sl\frac{\rho_s}{\rho_w})}{1 - \alpha(1 - sl\frac{\rho_s}{\rho_w})} \quad . \tag{11a}$$

Until now the boiling boundary has been considered constant. To include the effect of a variation of the boiling boundary we must notice that such a variation is equivalent to a source term at the position x which is delayed the time it takes a steam bubble to travel from the boiling boundary to the position x, i. e.

$$\text{delay time} = \int_{H1+BB}^{x} \frac{dx'}{v_s(x', o)} \quad .$$

When this source term is introduced on the right-hand side of eq. (5) instead of that which has already been accounted for, the following relation is obtained:

$$\frac{1}{v_s(x,o)} \frac{\partial}{\partial t}\left\{v_s(x,o)\,\alpha(x,t)\right\} + \frac{\partial}{\partial x}\left\{v_s(x,o)\,\delta\alpha(x,t)\right\} =$$

$$-\frac{q(BB,o)}{r\,\rho_s}\,\delta(x)\cdot\,\delta BB\left(t-\int_{H1+BB}^{x}\frac{dx'}{v_s(x',o)}\right). \qquad (12)$$

Here $\delta(x)$ denotes the delta function.

Laplace transformation and integration of eq. (12) yield

$$v_s(x,o)\,\overline{\delta\alpha}(x,s) = e^{-s\int_{H1+BB}^{x}\frac{dx''}{v_s(x'',o)}}\cdot\,\overline{\delta BB}(s)$$

$$\left\{-\int_{BB+H1}^{x}\frac{q(BB,o)\,\delta(x)}{r\,\rho_s}\,e^{-s\int_{H1+BB}^{x'}\frac{dx''}{v_s(x'',o)}}\,e^{s\int_{H1+BB}^{x'}\frac{dx''}{v_s(x'',o)}}\,dx'\right\}$$

$$= -e^{-s\int_{H1+BB}^{x}\frac{dx''}{v_s(x'',o)}}\,\frac{q(BB,o)}{r\,\rho_s}\,\overline{\delta BB}(s)\,. \qquad (13)$$

To obtain the average void variation $\overline{\delta\alpha}(s)$ over the whole channel, we integrate eq. (13) over the boiling region by means of eq. (9) and divide it by H:

$$\overline{\delta\alpha}(s) = \frac{1}{H}\,\frac{q(BB,o)}{r\,\rho_s}$$

$$\frac{1}{s}\left(e^{\frac{s\cdot r\rho_s}{(1-sl\,\frac{\rho_s}{\rho_w})q_m}\ln\frac{1}{1+(1-sl\,\frac{\rho_s}{\rho_w})\frac{q_m\,HB}{r\,\rho_s\,sl\cdot v_{inlet}}}}-1\right)\overline{\delta BB}(s)\,. \qquad (14)$$

In deriving eq. (2), the first term on the right-hand side of eq. (1) was neglected. The importance of this term will now be given special consideration.

The term $v_w \frac{\partial T}{\partial x}$ is still assumed negligible, the saturation temperature being almost uniform throughout the boiling region.

Thus the additional contribution to the void fraction due to temperature variations in the boiling region may be calculated from eq. (13) by substituting $-\frac{(1-a(x,o))\,\varrho_w\,c}{r\,\varrho_s} \cdot s\,\overline{\delta T(s)}$ for the right-hand side.

The factor $(1-a(x,o))$ is obtained from eq. (8), with the approximation $\varrho_s << \varrho_w$:

$$1 - a(x,o) \sim \frac{sl \cdot v_{inlet}}{v_s(x,o)} .$$

Now the calculations proceed in the same way as above, leading to the result

$$\overline{\delta a(s)} = -\frac{sl \cdot v_{inlet} \cdot \varrho_w\, c}{q_m(1-sl\frac{\varrho_s}{\varrho_w}) \cdot H} \ln\left(1+\frac{HB\, q_m(1-sl\frac{\varrho_s}{\varrho_w})}{r\,\varrho_s \cdot sl \cdot v_{inlet}}\right) \cdot \overline{\delta T(s)}$$

$$\cdot \left\{ 1 + \frac{q_m\,(1-sl\frac{\varrho_s}{\varrho_w})}{s \cdot r\,\varrho_s \ln\left(1+\frac{HB\, q_m(1-sl\frac{\varrho_s}{\varrho_w})}{r\,\varrho_s \cdot sl \cdot v_{inlet}}\right)} \right.$$

$$\left. -s\,\frac{r\,\varrho_s}{q_m(1-sl\frac{\varrho_s}{\varrho_w})} \ln\left(1+\frac{HB\, q_m(1-sl\frac{\varrho_s}{\varrho_w})}{r\,\varrho_s\, sl \cdot v_{inlet}}\right) \right.$$

$$\left. (e \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -1) \right\} \quad . \quad (15)$$

From eqs. (11), (14) and (15) the transfer functions A_q, A_v, A_b and A_t are immediately derived:

$$A_q = \frac{C_{q1}}{1+as} + \frac{C_{q2}}{s(1+as)} + \frac{C_{q3}}{s(1+as)} e^{s\tau} \tag{16}$$

$$A_v = \frac{C_{v1}}{1+as} + \frac{C_{v2}}{s(1+as)} + \frac{C_{v3}}{s(1+as)} e^{s\tau} \tag{17}$$

$$A_b = \frac{C_{b1}}{s} + \frac{C_{b2}}{s} e^{s\tau} \tag{18}$$

$$A_t = C \frac{1+s\tau}{s\tau} - \frac{C_t}{s\tau} e^{s\tau} , \tag{19}$$

where

$$C_{q1} = \frac{\frac{HB}{H}}{1-sl\frac{\varrho_s}{\varrho_w}} \cdot \frac{1}{q_o} \tag{20}$$

$$C_{q2} = - C_{q1} \frac{sl \cdot v_{inlet}}{HB} \tag{21}$$

$$C_{q3} = - C_{q2} \tag{22}$$

$$C_{v1} = - C_{q1} \cdot \frac{q_o}{v_{inlet}} \tag{23}$$

$$C_{v2} = - C_{q2} \cdot \frac{q_o}{v_{inlet}} \tag{24}$$

$$C_{v3} = - C_{q3} \cdot \frac{q_o}{v_{inlet}} \tag{25}$$

$$C_{b1} = - \frac{q(BB, o)}{r \varrho_s} \frac{1}{H} \tag{26}$$

$$C_{b2} = - C_{b1} \tag{27}$$

$$C_t = \frac{sl \cdot v_{inlet} \cdot \varrho_w \cdot c \cdot \tau}{r \varrho_s H(1-sl \cdot \frac{\varrho_s}{\varrho_w})} \tag{28}$$

$$V_a = \frac{v_{inlet}(1-sl\frac{\varrho_s}{\varrho_w})}{1-a(1-sl\frac{\varrho_s}{\varrho_w})} \tag{28a}$$

$$a = \frac{r \varrho_s}{(1-sl\frac{\varrho_s}{\varrho_w}) q_m} \tag{29}$$

$$\tau = a \ln \frac{1}{1 + \frac{HB}{a \cdot sl \cdot v_{inlet}}} \quad . \tag{30}$$

(b) Transfer functions B_q, B_v, B_p, B_t

These transfer functions determine the variation of the boiling boundary with the variations of heat flux in the subcooled region, coolant inlet velocity, system pressure, and coolant inlet temperature.

Eq. (1), which is valid both in the subcooled and in the boiling region, yields, when we put $a \equiv 0$,

$$\frac{\partial T(x,t)}{\partial t} + v_w \frac{\partial T(x,t)}{\partial x} = \frac{q(x,t)}{\varrho_w c} \quad . \tag{31}$$

Introducing the variations of T and q, we have

$$\frac{\partial}{\partial t} \delta T(x,t) + v_w \frac{\partial}{\partial x} \delta T(x,t) = \frac{\delta q(x,t)}{\varrho_w c} \quad . \tag{32}$$

Noting that v_w is spatially constant in the subcooled region, we obtain by Laplace transformation and integration

$$\overline{\delta T}(x,s) = e^{-\frac{s}{v_w}(x-H1)} \left\{ \frac{1}{v_w \varrho_w c} \int_{H1}^{x} \overline{\delta q}(x',s) e^{\frac{s}{v_w}(x'-H1)} dx' + \overline{\delta T_{inlet}}(s) \right\}. \tag{33}$$

Assuming q(x, t) to be separable in space and time and putting $q(x,o) = q_o \sin(\frac{x}{L}\pi)$, we may perform the integration in eq. (30):

$$\overline{\delta T}(x, s) = e^{-\frac{sx}{v_w}} \left\{ \frac{q_o}{v_w \rho_w c} \frac{\overline{\delta q_o}(s)}{q_o} \left[\frac{\frac{s}{v_w}\sin(\frac{x'\pi}{L}) - \frac{\pi}{L}\cos(\frac{x'\pi}{L})}{\frac{s^2}{v_w^2} + \frac{\pi^2}{L^2}} e^{\frac{s}{v_w}x} \right]_{H1}^{x} \right.$$

$$\left. + \overline{\delta T}_{inlet}(s) \right\} \qquad (34)$$

$$\overline{\delta T}(H1+BB, s) = e^{-s\frac{BB}{v_w}} \left\{ \frac{q_o}{v_w \rho_w c} \frac{\overline{\delta q_o}(s)}{q_o} \right.$$

$$\left\{ \frac{sv_w \sin(\frac{H1+BB}{L}\pi) - \frac{v_w^2\pi}{L}\cos(\frac{H1+BB}{L}\pi)}{s^2 + \frac{\pi^2}{L^2}v_w^2} e^{s\frac{BB}{v_w}} \right.$$

$$\left. \left. - \frac{sv_w \sin(\frac{H1}{L}\pi) - \frac{v_w^2\pi}{L}\cos(\frac{H1}{L}\pi)}{s^2 + \frac{\pi^2}{L^2}v_w^2} \right\} + \overline{\delta T}_{inlet}(s) \right\}. \qquad (35)$$

Until now the coolant inlet velocity has been assumed constant. Introducing $v_w(t) = v_w(o) + \delta v_w(t)$ in eq. (28), we have

$$\frac{\partial}{\partial t}T(x, t) + v_w(o)\frac{\partial}{\partial x}T(x, t) = \frac{q(x, t)}{\rho_w \cdot c} - \delta v_w(t)\frac{\partial}{\partial x}T(x, t)$$

$$\sim \frac{q(x, t)}{\rho_w \cdot c}\left(1 - \frac{\delta v_w(t)}{v_w(o)}\right), \qquad (36)$$

which shows that a variation in the coolant velocity is approximately equivalent to a heat-flux variation of the magnitude $-q(x, t)\frac{\delta v_w}{v_w}$.

Insertion of this result into eq. (35) yields

$$\overline{\delta T}(BB, s) = e^{-s\frac{BB}{v_w}} \frac{q_o}{v_w \rho_w c}\left\{ \frac{\overline{\delta q_o}(s)}{q_o} - \frac{\overline{\delta v}_{inlet}(s)}{v_{inlet}} \right\}.$$

$$\left\{ - \frac{1-s\,\frac{L}{\pi v_w}\,\mathrm{tg}\left(\frac{H1+BB}{L}\pi\right)}{1+s^2\,\frac{L^2}{\pi^2 v_w^2}}\,\frac{L}{\pi}\cos\left(\frac{H1+BB}{L}\pi\right)\,e^{s\frac{BB}{v_w}} \right.$$

$$\left. + \frac{1-s\,\frac{L}{\pi v_w}\,\mathrm{tg}\left(\frac{H1}{L}\pi\right)}{1+s^2\,\frac{L^2}{\pi^2 v_w^2}}\,\frac{L}{\pi}\cos\left(\frac{H1}{L}\pi\right) \right\}$$

$$+ e^{-s\frac{BB}{v_w}}\,\overline{\delta T_{inlet}(s)} \quad . \tag{37}$$

When the relationship

$$\overline{\delta BB(s)} = - \frac{\partial x}{\partial T}\left\{ \overline{\delta T(BB, s)} - \frac{\partial T_{sat}}{\partial p}\,\overline{\delta p(s)} \right\} \tag{38}$$

$$\text{with } \frac{\partial x}{\partial T} \sim \frac{v_w \rho_w c}{q(BB, o)}$$

and eq. (37) are used, it is now possible to write down the desired transfer functions:

$$B_q = D_{q1}\,\frac{1+b_1 s}{1+b_2 s^2} + D_{q2}\,\frac{1+b_3 s}{1+b_2 s^2}\,e^{s\tau} \tag{39}$$

$$B_v = D_{v1}\,\frac{1+b_1 s}{1+b_2 s^2} + D_{v2}\,\frac{1+b_3 s}{1+b_2 s^2}\,e^{s\tau} \tag{40}$$

$$B_p = D_p \tag{41}$$

$$B_t = D_t\,e^{s\tau}\,, \tag{42}$$

where

$$D_{q1} = \frac{1}{q(BB, o)}\,\frac{L}{\pi}\cos\left(\frac{H1+BB}{L}\pi\right) \tag{43}$$

$$D_{q2} = - D_{q1} \frac{\cos(\frac{H1}{L}\pi)}{\cos(\frac{H1+BB}{L}\pi)} \tag{44}$$

$$D_{v1} = - D_{q1} \frac{q_o}{v_{inlet}} \tag{45}$$

$$D_{v2} = - D_{q2} \frac{q_o}{v_{inlet}} \tag{46}$$

$$D_p = \frac{v_{inlet} \rho_w c}{q(BB, o)} \quad \frac{\partial T_{sat}}{\partial p} \tag{47}$$

$$D_t = - \frac{v_{inlet} \rho_w c}{q(BB, o)} \tag{48}$$

$$b_1 = - \frac{L}{\pi v_{inlet}} \text{ tg } (\frac{H1+BB}{L}\pi) \tag{49}$$

$$b_2 = \frac{L^2}{\pi^2 v_{inlet}^2} \tag{50}$$

$$b_3 = - \frac{L}{\pi v_{inlet}} \text{ tg } (\frac{H1}{L}\pi) \tag{51}$$

$$\tau = - \frac{BB}{v_{inlet}} \quad . \tag{52}$$

(c) Transfer functions g, c_{sat}, c_{sub}, b_{sat}, b_{sub}, b_a

These transfer functions are all constants which can be immediately derived:

The effect of g is to provide a weighting between the two heat-flux contributions q_{sat} and q_{sub}, which correspond to the boiling and the subcooled region respectively. The weighting should be in accordance with the sinusoidal spatial heat-flux distribution; hence

$$g = \frac{\int_{H1}^{H1+BB} \sin(\frac{x}{L}\pi)dx}{\int_{H1}^{L-Hn} \sin(\frac{x}{L}\pi)dx} = \frac{\cos(\frac{\pi}{L}H1) - \cos(\frac{\pi}{L}(H1+BB))}{\cos(\frac{\pi}{L}H1) - \cos(\frac{\pi}{L}(L-Hn))} \quad . \qquad (53)$$

The effect of c_{sat} and c_{sub} is to provide a similar weighting of the temperature variations in the boiling and subcooled regions regarding the reactivity feedback through the coolant temperature. This weighting is based on the coolant masses of the two regions; hence

$$c_{sub} = \frac{\varrho_w \cdot BB}{\varrho_w \cdot BB + (a\,\varrho_s + (1-a)\,\varrho_w)(L-H1-Hn-BB)} \qquad (54)$$

$$c_{sat} = 1 - c_{sub} \quad . \qquad (55)$$

b_{sat}, b_{sub} and b_a account for the effect on the mean coolant density of the temperatures of the boiling and subcooled regions and of the void variation. Their values are readily found:

$$b_{sat} = \frac{\partial \varrho_w}{\partial T}\left(\frac{L-H1-Hn-BB}{L-H1-Hn} - a\right) + \frac{\partial \varrho_s}{\partial T}\,a \qquad (56)$$

$$b_{sub} = \frac{\partial \varrho_w}{\partial T}\,\frac{BB}{L-H1-Hn} \qquad (57)$$

$$b_a = (\varrho_s - \varrho_w)\,\frac{\frac{1}{2} - \frac{L}{4\pi HB}\left(\sin\frac{2\pi}{L}Hn - \sin\frac{2\pi}{L}(H1+BB)\right)}{\frac{1}{2} - \frac{L}{4\pi H}\left(\sin\frac{2\pi}{L}Hn - \sin\frac{2\pi}{L}H1\right)} \quad . \qquad (58)$$

In formula (58) regard has been paid to the relative importance of the location of the steam voids by weighting with the flux shape function raised to the second power and assuming the voids evenly distributed over the boiling region.

(d) Transfer functions Y_t, Y_m, Y_c, d_c

These are all reactivity feedback coefficients. They may be found from theoretical lattice calculations, from measurements or from computer runs. For the purpose of the application of section 3, the last method was chosen, and hence no formulae are given here.

(e) Transfer functions Y_p, Y_t, X_p, X_q

These transfer functions describe the heat release from the fuel. A derivation of their exact form and a discussion of some approximative expressions are found in ref. 7, chapter II, section 3. From this reference the following is obtained:

$$Y_p \cong \frac{nr \cdot r_a^2}{r_i^2 - nr \cdot r_c^2} \cdot \frac{1 + a_1(\varepsilon)\frac{1}{4}\frac{r_a^2}{\kappa_1}\rho_1 c_1 s + a_2(\varepsilon)\left(\frac{1}{4}\frac{r_a^2}{\kappa_1}\rho_1 c_1\right)^2 s^2}{1 + b_1(\varepsilon)\frac{1}{4}\frac{r_a^2}{\kappa_1}\rho_1 c_1 s + b_2(\varepsilon)\left(\frac{1}{4}\frac{r_a^2}{\kappa_1}\rho_1 c_1\right)^2 s^2 + b_3(\varepsilon)\left(\frac{1}{4}\frac{r_a^2}{\kappa_1}\rho_1 c_1\right)^3 s^3} \tag{59}$$

$$\varepsilon = \frac{1}{r_a^2 \rho_1 c_1}\left\{\rho_2 c_2 \frac{r_b}{r_c}\frac{\Delta r}{k} + \frac{1}{2}\frac{r_a^2}{r_c}\rho_1 c_1\left(\frac{1}{k} + \frac{\Delta r}{\kappa_2} + \eta\frac{r_c}{r_b}\right)\right\} \tag{60}$$

$$Y_t \cong -\rho_1 c_1 \left(1 + \frac{2r_b}{r_a^2} \cdot \frac{\rho_2 c_2}{\rho_1 c_1}\Delta r\right) s\, Y_p \tag{61}$$

$$X_p \cong \frac{1}{\rho_1 c_1\left(1 + \frac{2r_b}{r_a^2}\frac{\rho_2 c_2}{\rho_1 c_1}\Delta r\right)} \cdot \frac{1}{s} \tag{62}$$

$$X_q \cong -X_p \frac{r_i^2 - nr \cdot r_c^2}{nr \cdot r_a^2} . \tag{63}$$

Here, the coefficients $a_1(\varepsilon)$, $b_1(\varepsilon)$, and $b_2(\varepsilon)$ are obtained from fig. 10 of ref. 7.

The nomenclature is the same as in ref. 7. Further, nr denotes the number of rods in a fuel element and r_i the inner radius of the shroud. We also note that the real factor in eqs. (56) and (60) differs from that obtained

in ref. 7. This stems from the fact that ref. 7 operates with the heat flux per unit surface area of the fuel while here the heat flow to the coolant is taken per unit volume of coolant.

(f) Transfer function Z_o

This is the well-known zero-power transfer function of the reactor which governs the kinetic behaviour of the neutron flux. In the one-delayed-group approximation this function has the form

$$Z_o = \frac{P(o)\lambda}{l\lambda+\beta} \cdot \frac{1}{s} \cdot \frac{1+\frac{1}{\lambda}s}{1+\frac{1}{l\lambda+\beta}s} \quad . \tag{64}$$

Here λ denotes the decay constant and β the relative magnitude of the delayed neutron group, while l is the neutron lifetime.

3. A Worked Example

Consider a design with

$L = 4.40$ m

$Hl = 0.30$ m

$Hn = 0.60$ m

$p = 70$ at

$v_{inlet} = 1.02$ m/sec

$T_{inlet} = 547.5\ ^oK$

$sl = 2$

$r_a = 0.006350$ m

$r_b = 0.006475$ m

$r_c = 0.007025$ m

$\varrho_1 = 10900$ kg/m^3

$c_1 = 0.238$ k Joule/kg/^{o}K

$\varkappa_1 = 0.003$ kWatt/m/^{o}K

$\varrho_2 = 6570$ kg/m^3

$c_2 = 0.297$ k Joule/kg/^{o}K

$\varkappa_2$ = 0.0125kWatt/m/^oK

η = 0.05 (kWatt/m^2/^oK)$^{-1}$

r_i = 0.074 m

nr = 36

λ = 0.076 sec^{-1}

β = 0.00755.

The details of the lattice structure are not specified here. The only quantities derived from this structure have been taken over directly from computer runs with a code called NEUPHTEST, which is mentioned in ref. 7. These quantities are:

l = $5.6 \cdot 10^{-4}$ sec

γ_f = $-2.3 \cdot 10^{-5}$ $^oK^{-1}$

γ_m = $-3.8 \cdot 10^{-4}$ $^oK^{-1}$

γ_c = $-2.1 \cdot 10^{-5}$ $^oK^{-1}$

d_c = $1.81 \cdot 10^{-5}$ m^3/kg.

The power level, which can be specified arbitrarily, is fixed as

$$P(x, o) = P_o \sin\left(\frac{x}{L}\pi\right)$$

with

P_o = $3.16 \cdot 10^5$ kWatt/m^3.

Hence the heat flux is

$$q(x, o) = q_o \sin\left(\frac{x}{L}\pi\right)$$

with

q_o = $1.23 \cdot 10^5$ kWatt/m^3.

Computer runs with the code BRENDA, as described in ref. 7, now yield the average coolant temperature

T_c = 557.6 ^oK,

the length of the subcooled region

BB = 0.63 m

and the average void fraction

$\alpha = 0.46.$

From heavy-water thermodynamic tables it appears that

$\varrho_s(p) = 41.16\ kg/m^3$

$\varrho_w(T_c) = 811.8\ kg/m^3$

$r(p) = 1357\ k\ Joule/kg$

$c(T_c) = 5.26\ k\ Joule/kg/^oK$

$\frac{\delta T_{sat}}{\delta p}(p) = 0.963\ ^oK/at$

$\frac{\delta \varrho_w}{\delta T}(T_c) = -2.14\ kg/m^3/^oK$

$\frac{\delta \varrho_s}{\delta T}(p) = 0.695\ kg/m^3/^oK\ .$

The quantity k , the heat conductivity of the interface between fuel and coolant, has to be deduced from the above:

The heat-transfer equation has the form

$q = c(T_f - T_c)^4$ (Mc Adam's formula, ref. 9).

For small deviations from the steady state the differential formula

$$\delta q = 4\, c\, (T_f - T_c)^3\ \ \delta(T_f - T_c) = k\, \delta(T_f - T_c)$$

applies.

Hence

$$k = 4\, c\, (T_f - T_c)^3 = 4\, \frac{q}{T_f - T_c}\ .$$

From computer runs with BRENDA, corresponding values of q and $T_f - T_c$ have been deduced, yielding

$k = 54\ kWatt/m^2/^oK.$

Further, the mean heat flux in the boiling region and the heat flux at the boiling boundary are of interest:

$q_m = 9.45 \cdot 10^4\ kWatt/m^3$

$q(BB) = 1.12 \cdot 10^5\ kWatt/m^3\ .$

Calculation by formulae (20) to (30) and (43) to (58) yields

$C_{q1} = 7.42 \cdot 10^{-6}$ (kWatt/m^3)$^{-1}$

$C_{q2} = -5.27 \cdot 10^{-6}$ (k Joule/m^3)$^{-1}$

$C_{q3} = 5.27 \cdot 10^{-6}$ (k Joule/m^3)$^{-1}$

$C_{v1} = -0.895$ (m/sec)$^{-1}$

$C_{v2} = 0.635$ (m/sec)$^{-1}$

$C_{v3} = -0.635$ (m/sec)$^{-1}$

$C_{b1} = -0.573$ m^{-1} sec^{-1}

$C_{b2} = 0.573$ m^{-1} sec^{-1}

$C_t = -3.25 \cdot 10^{-2}$ $^oK^{-1}$

$a = 0.659$ sec

$\tau = -0.753$ sec.

$V_a = 1.560$ m/sec

$D_{q1} = 0.985 \cdot 10^{-5}$ m^4/kWatt

$D_{q2} = -1.22 \cdot 10^{-5}$ m^4/kWatt

$D_{v1} = -1.188$ sec

$D_{v2} = 1.47$ sec

$D_p = 3.75 \cdot 10^{-2}$ m/at

$D_t = -3.89 \cdot 10^{-2}$ m/^{o}K

$b_1 = -1.072$ sec

$b_2 = 1.883$ sec^2

$b_3 = -0.298$ sec

$\tau = -0.618$ sec

$g = 0.1$

$1-g = 0.9$

$c_{sub} = 0.279$

$c_{sat} = 0.721$

$b_{sat} = -0.462$ kg/m^3/^{o}K

$b_{sub} = -0.385$ kg/m^3/^{o}K

$b_a = -886.3\ kg/m^3$.

Formula (60) yields

$\varepsilon = 0.106$.

Then, from ref. 7, fig. 10:

$a_1(\varepsilon) = 0.250$

$a_2(\varepsilon) = 0.00620$

$b_1(\varepsilon) = 0.850$

$b_2(\varepsilon) = 0.080$

$b_3(\varepsilon) = 0.00090$.

Now, from eq. (59):

$$Y_p = 0.392\ \frac{1 + 2.18\ s + 0.473\ s^2}{1 + 7.42\ s + 6.10\ s^2 + 0.598\ s^3}\ ,$$

and from eq. (61):

$$Y_t = -1466\ k\ Joule/m^3/^oK \cdot s\ \frac{1 + 2.18\ s + 0.473\ s^2}{1 + 7.42\ s + 6.10\ s^2 + 0.598\ s^3}\ .$$

Eq. (62) yields

$$X_p = 2.67 \cdot 10^{-4} \cdot \frac{1}{s}\ m^3\ ^oK/k\ Joule\ ,$$

and eq. (63):

$$X_q = -6.80 \cdot 10^{-4} \cdot \frac{1}{s}\ m^3\ ^oK/k\ Joule\ .$$

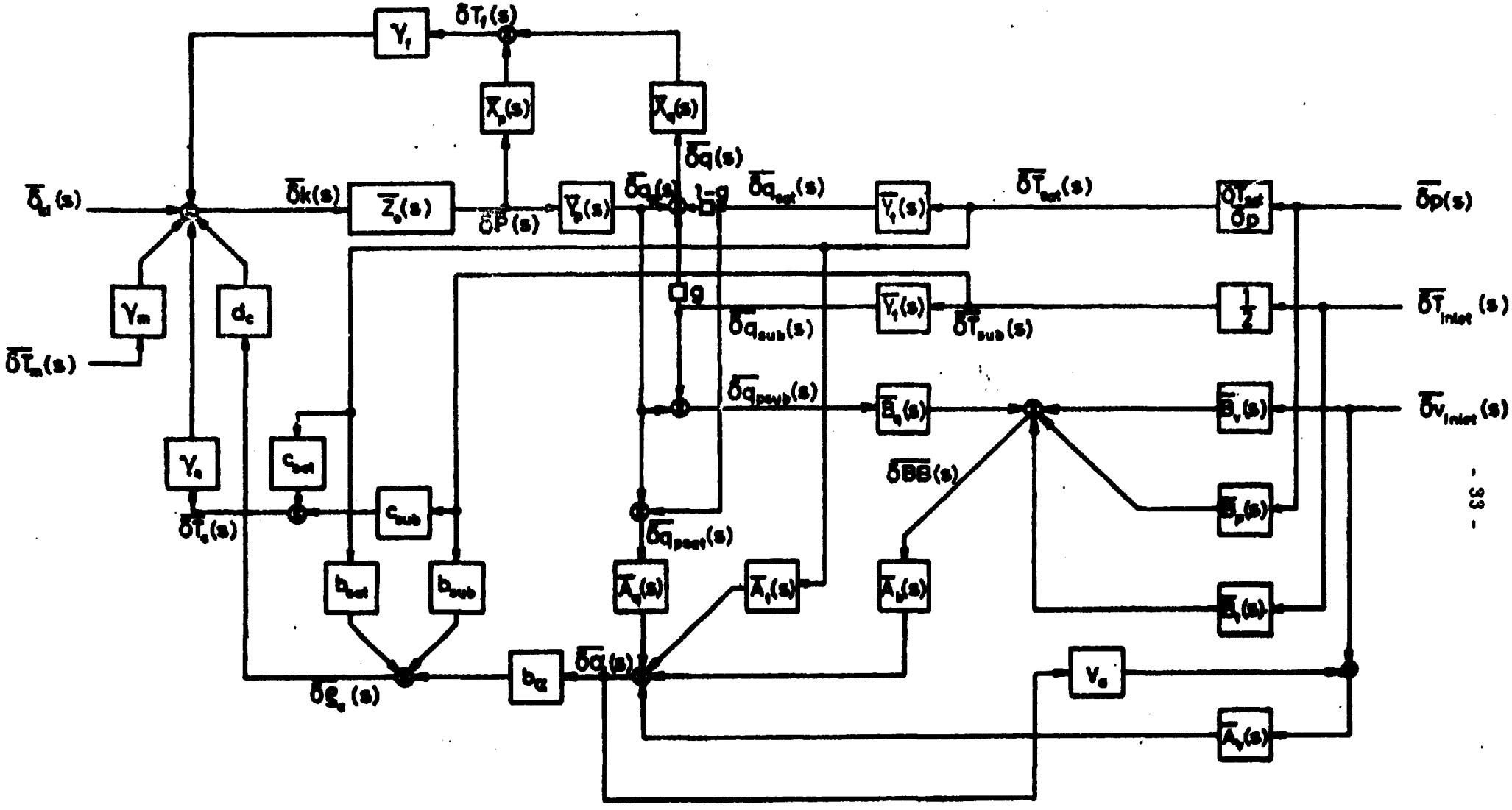
The zero-power transfer function is obtained from eq. (64):

$$Z_o = 3.16 \cdot 10^6\ kWatt/m^3/sec \cdot \frac{1}{s}\ \frac{1 + 13.15\ s}{1 + 0.0738\ s}\ .$$

At this stage all partial transfer functions of the model have been determined numerically and presented in a form which allows immediate use of a digital computer code named TRANSFER 1 (ref. 24) to find any desired transfer function between two arbitrarily chosen parameters. Some examples are shown in figs. 2-21.

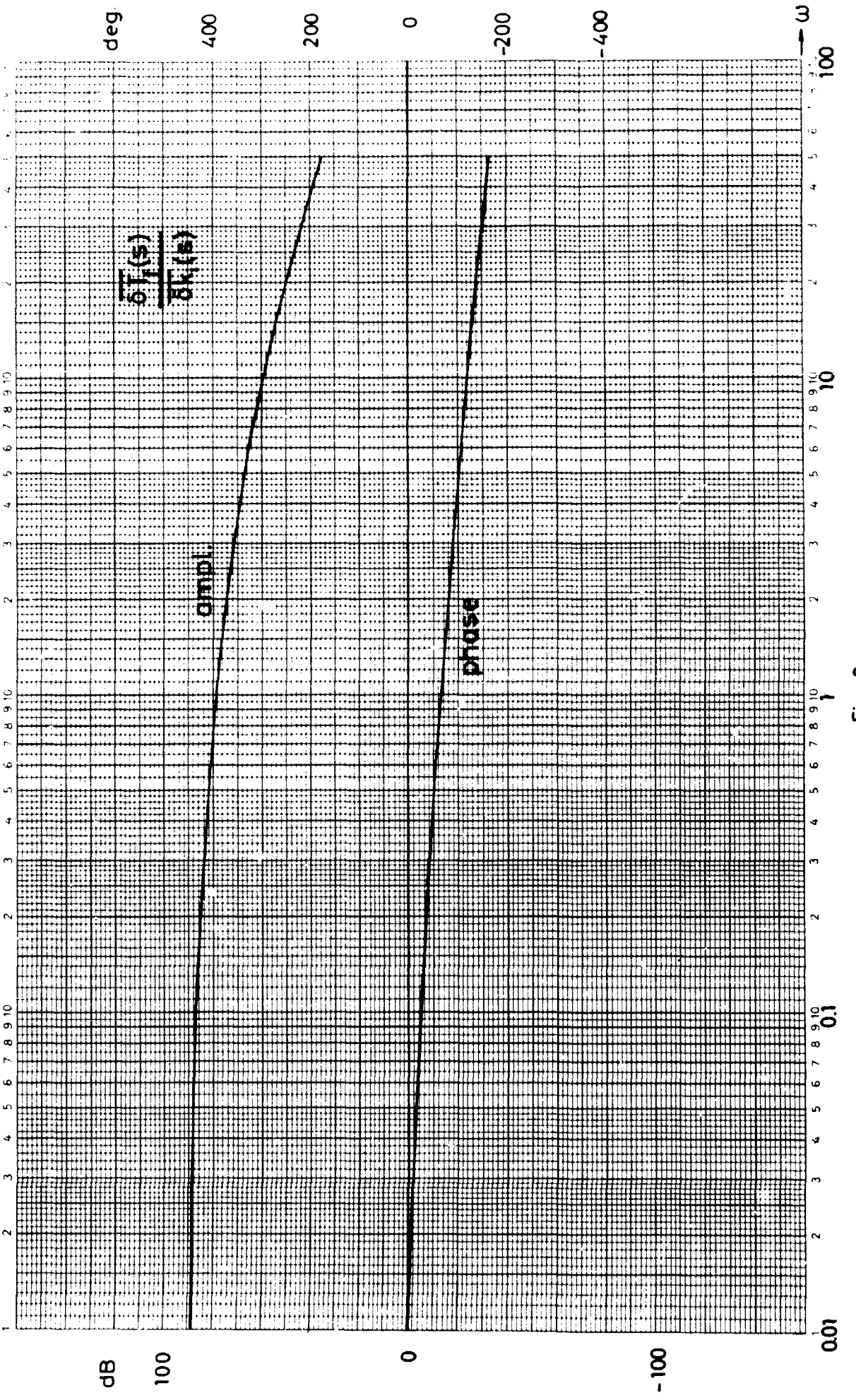
Table 1

δk_i	Reactivity variation controlled by control system.
δT_m	Moderator temperature variation, spatial average.
δp	System pressure variation.
δT_{inlet}	Coolant inlet temperature variation.
δv_{inlet}	Coolant inlet velocity variation.
δk	Total reactivity variation.
δT_c	Coolant temperature variation, average over whole channel.
δT_f	Fuel temperature variation, average over fuel volume.
$\delta \varrho_c$	Coolant density variation, average over whole channel.
δP	Power density variation, average over fuel volume.
δq_p	Variation of heat inflow into coolant caused by power variation; average over whole channel.
δq_{sat}	Variation of heat inflow into coolant caused by saturation temperautre variation, saturation region assumed extended over whole channel; average over whole channel.
δq_{sub}	Variation of heat inflow into coolant caused by subcooled temperature variation, subcooled region assumed extended over whole channel; average over whole channel.
$\delta q_{p\ sat}$	$\delta q_p + \delta q_{sat}$.
$\delta q_{p\ sub}$	$\delta q_p + \delta q_{sub}$.
δq	Actual variation of heat inflow into coolant, average over whole channel.
δa	Steam void variation, average over whole channel.
δT_{sat}	Saturation temperature variation.
δT_{sub}	Subcooled temperature variation, average over subcooled region.
δBB	Boiling boundary variation.



LINEAR DYNAMIC MODEL OF A COOLING CHANNEL
IN A BOILING-HEAVY-WATER REACTOR

Fig.1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400
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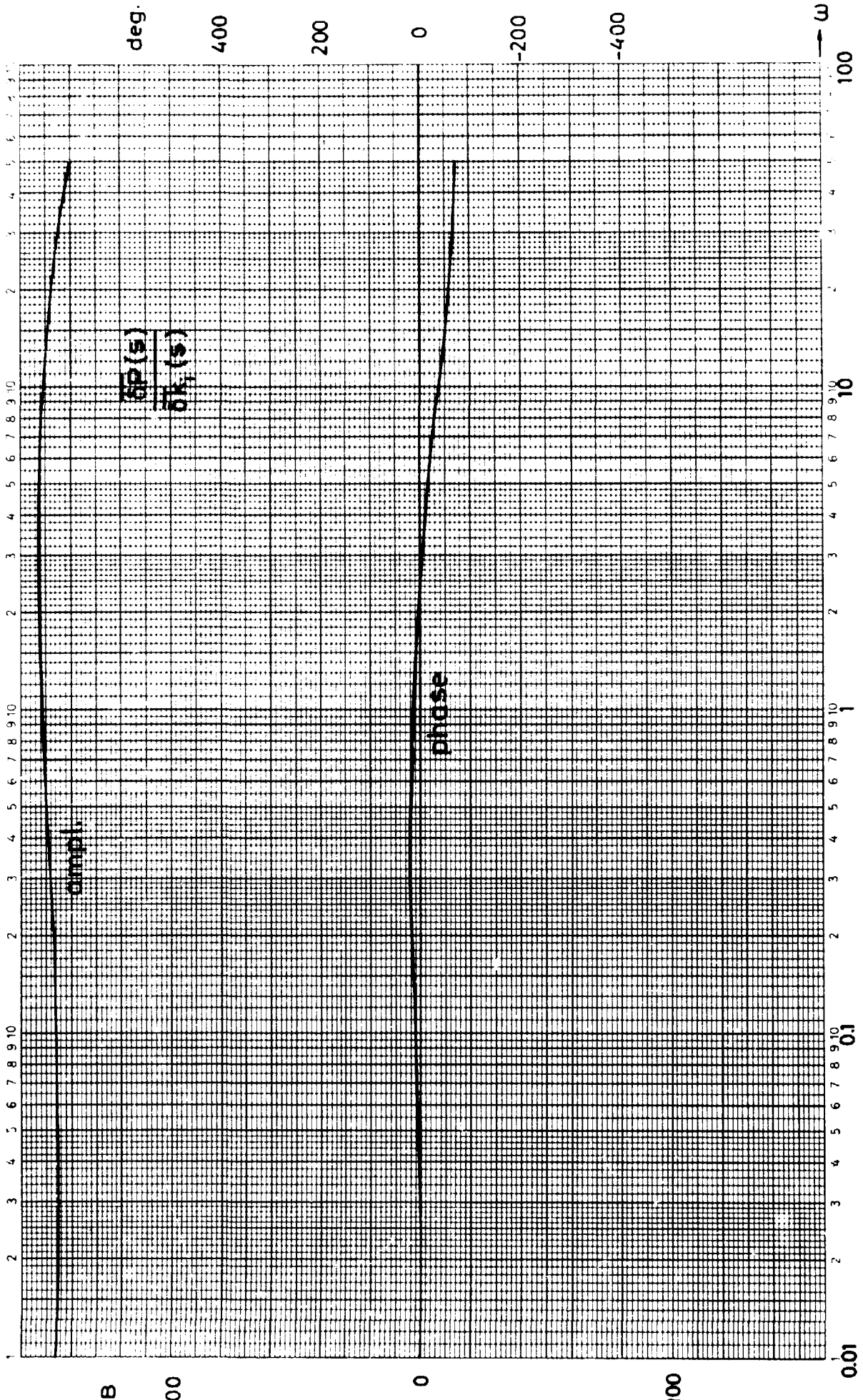
Fig. 2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ampl.
phase
$\frac{\delta P(s)}{\delta K_T(s)}$

Fig.3

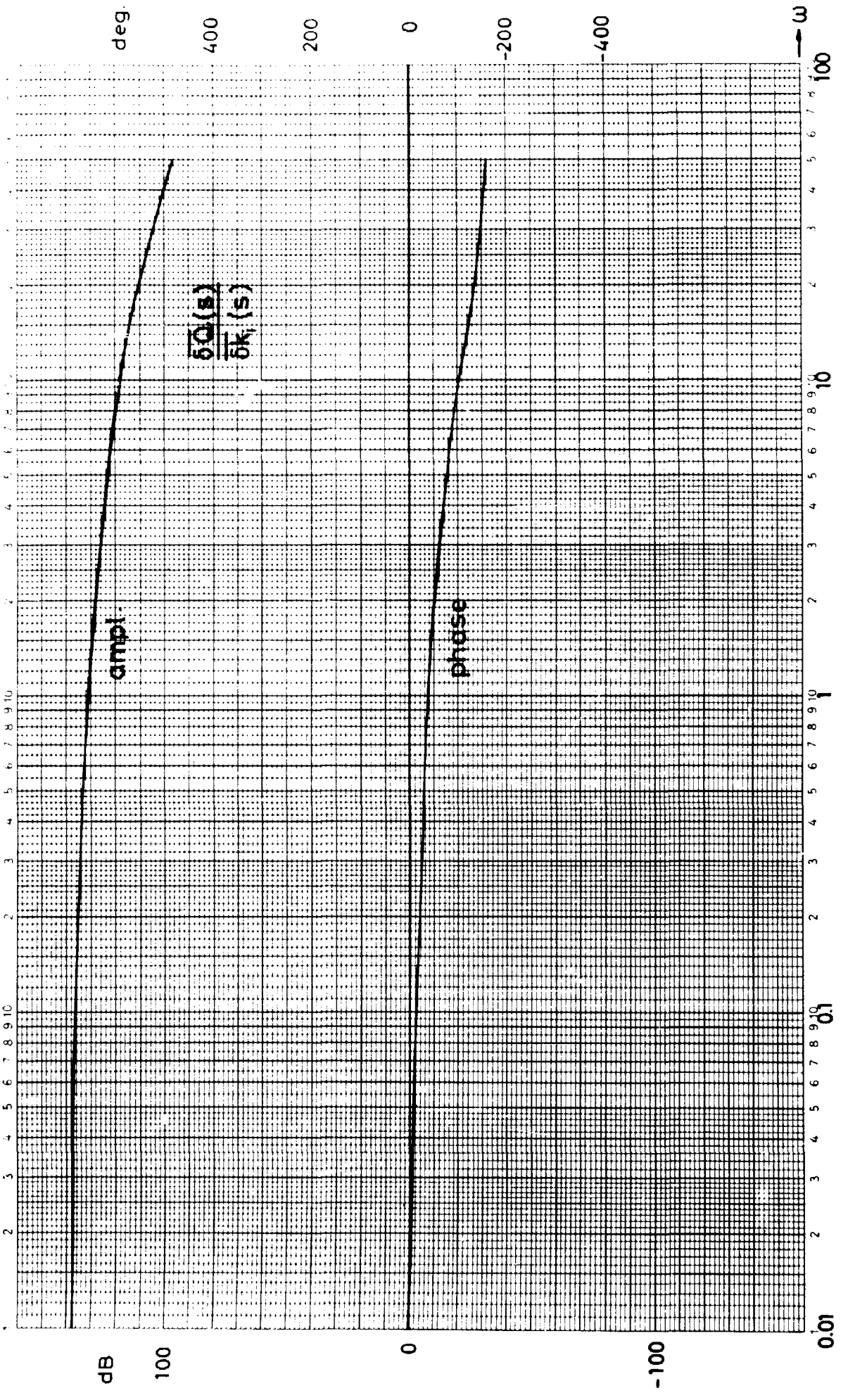
dB
100
0
-100
deg.
400
200
0
-200
-400
0.01
0.1
1
10
100
ω
ampl.
phase
$\frac{\delta \bar{Q}(s)}{\delta k_i(s)}$

Fig.4

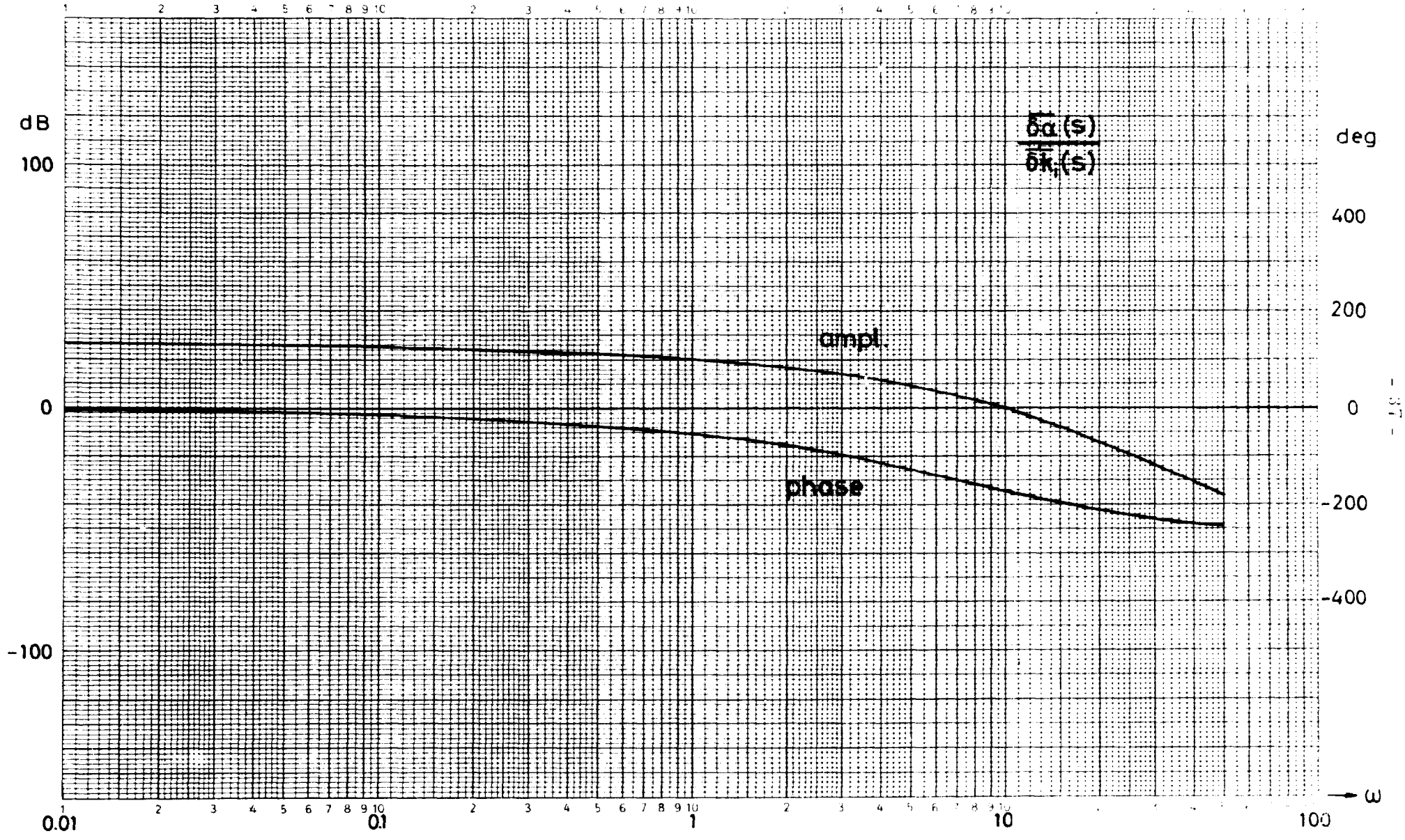
dB
100
0
-100
deg
400
200
0
-200
-400
ampl.
phase
$\frac{\delta\alpha(s)}{\delta k_i(s)}$
ω
0.01
0.1
1
10
100

Fig.5

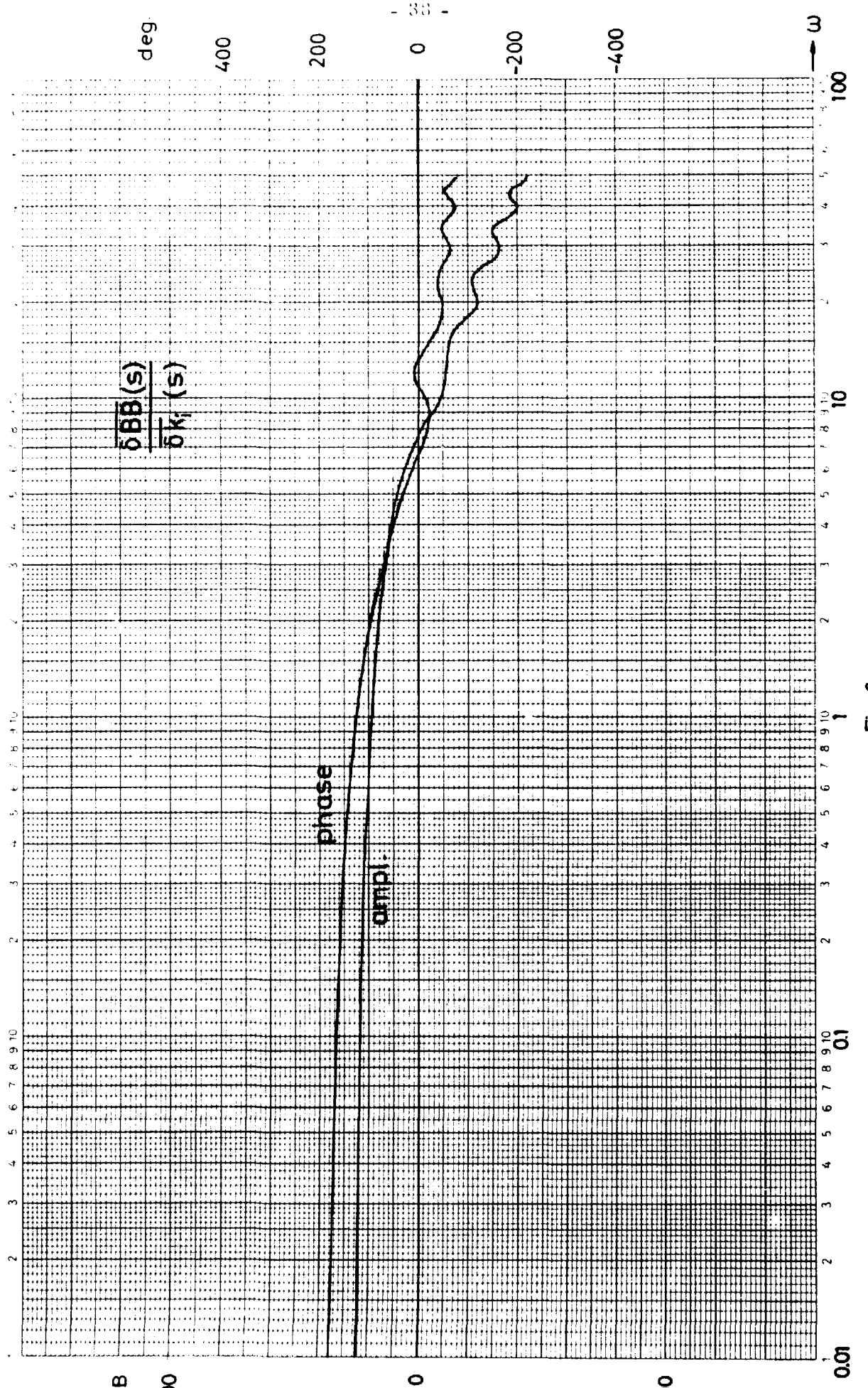
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
$\frac{\delta\overline{BB}(s)}{\delta\overline{k}_i(s)}$
phase
ampl.

Fig. 6

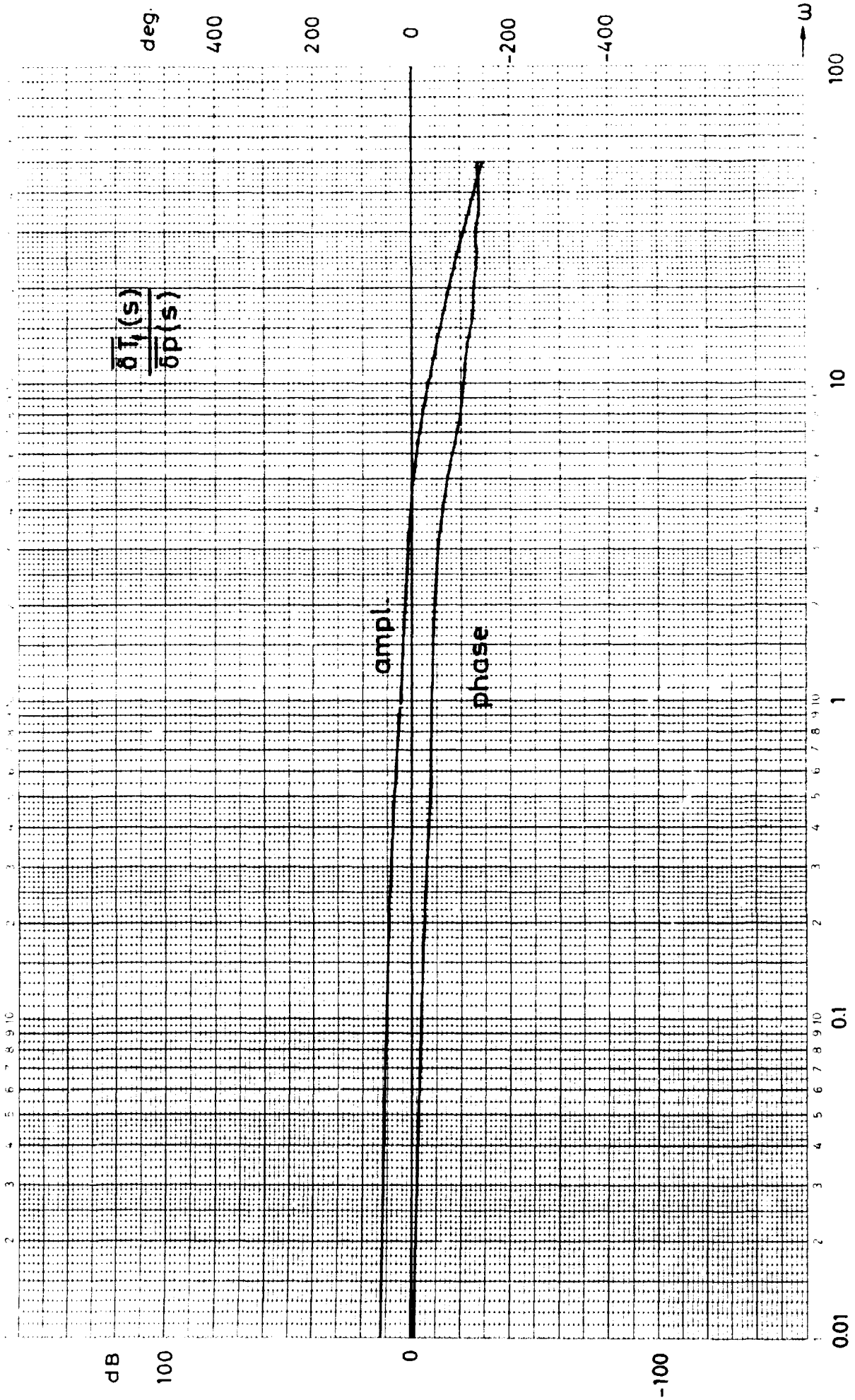
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
$\frac{\delta \bar{T}_1(s)}{\delta \bar{p}(s)}$
ampl
phase

Fig. 7

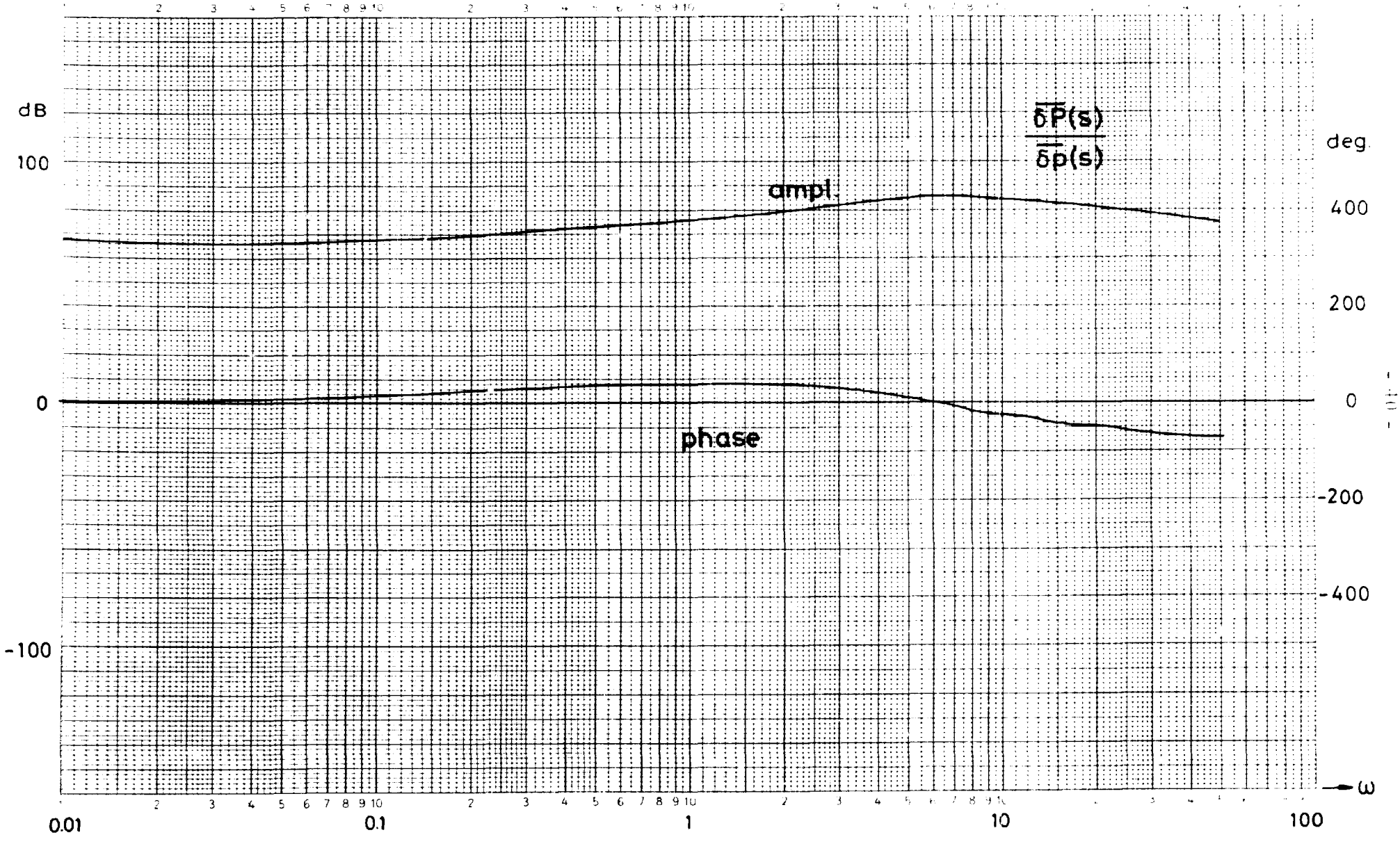
dB
100
0
-100
deg.
400
200
0
-200
-400
ampl.
phase
$\frac{\overline{\delta P(s)}}{\overline{\delta p(s)}}$
ω
0.01
0.1
1
10
100

Fig. 8

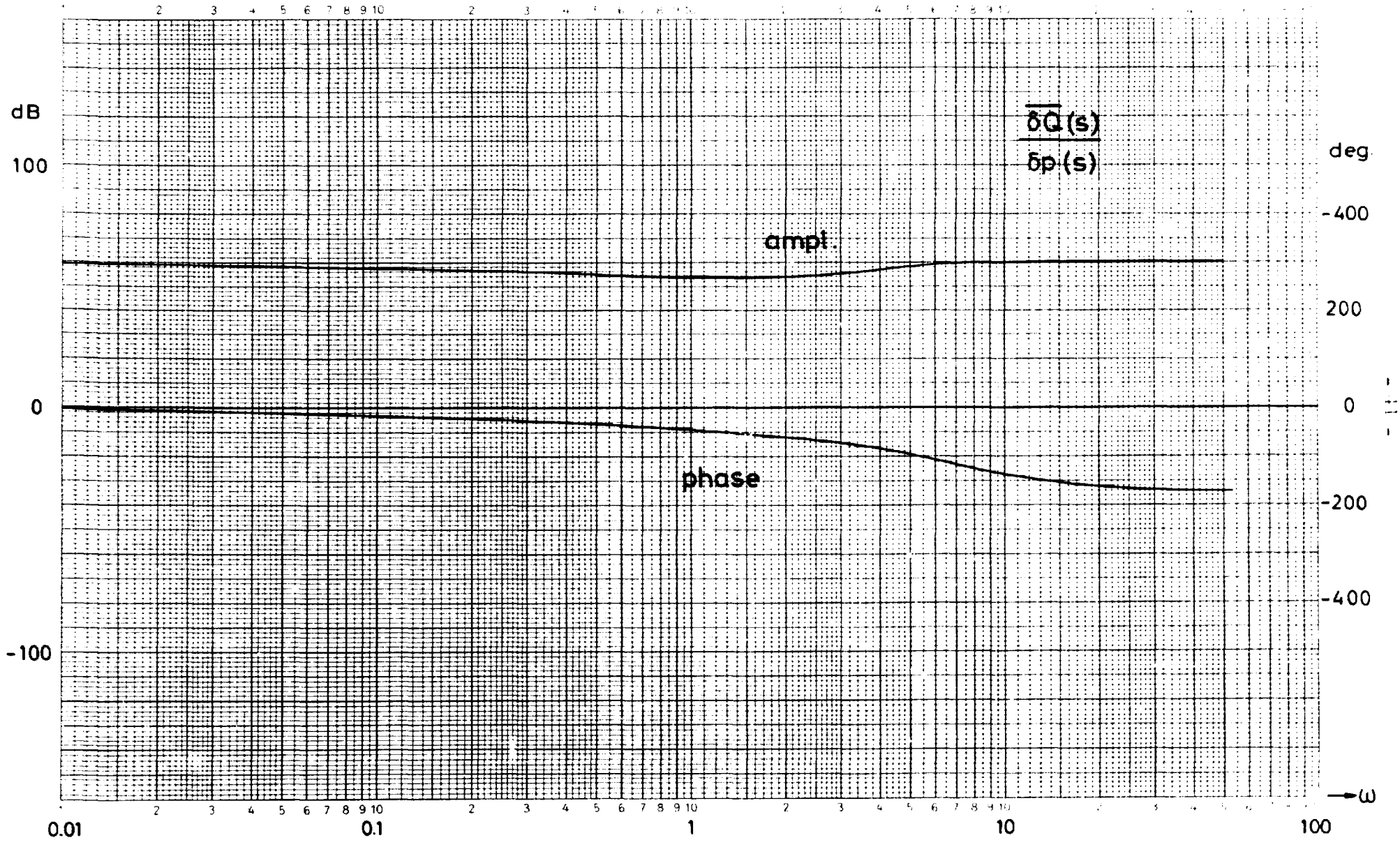
dB
100
0
-100
deg
-400
200
0
-200
-400
ampt.
phase
δQ(s)
δp(s)
ω
0.01
0.1
1
10
100

Fig.9

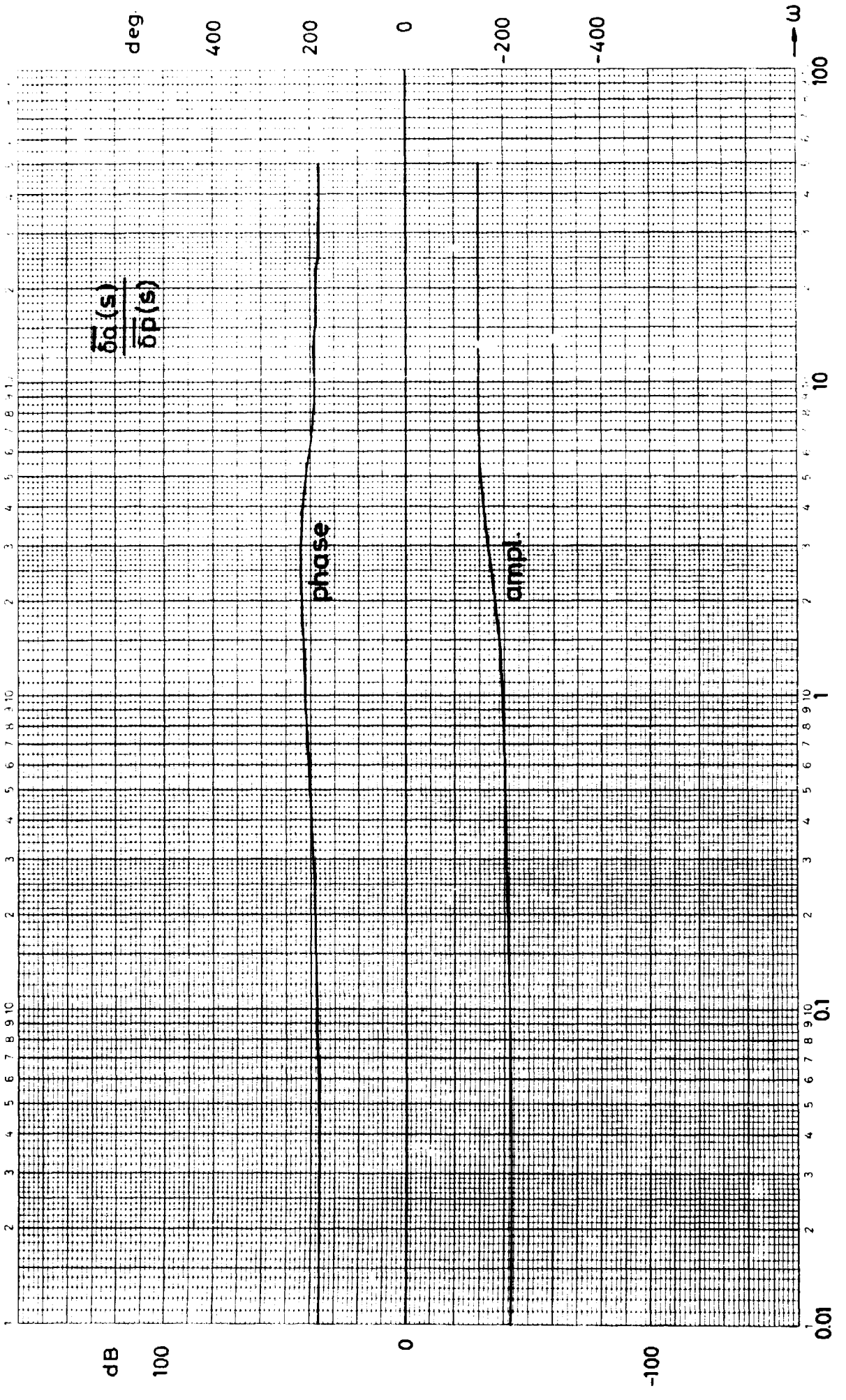
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
$\frac{\overline{\delta q}(s)}{\overline{\delta p}(s)}$
phase
ampl.

Fig.10

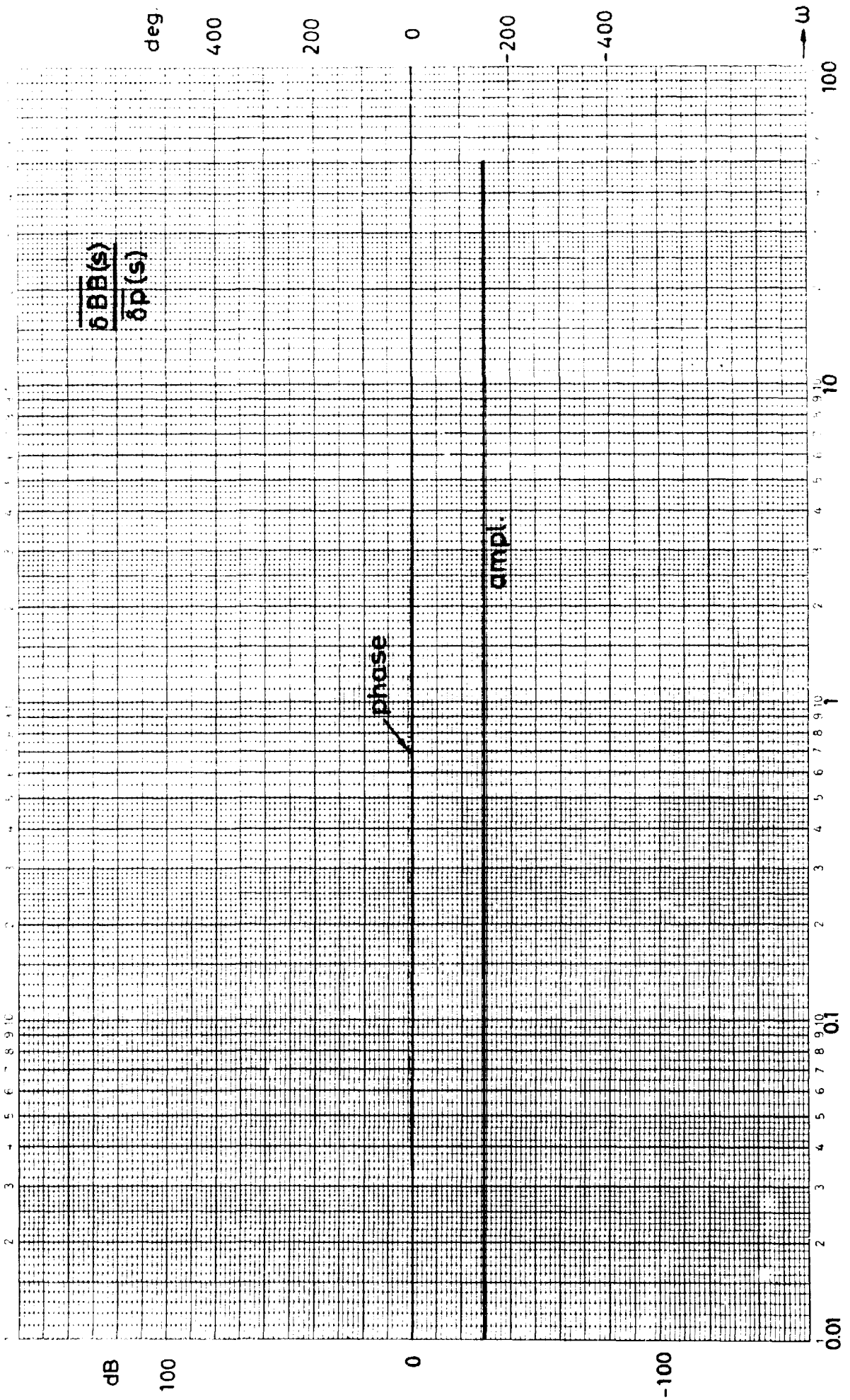
dB
100
0
-100
deg
400
200
0
-200
-400
ω
0.01
0.1
1
10
100
δBB(s)
δp(s)
phase
ampl.

Fig.11

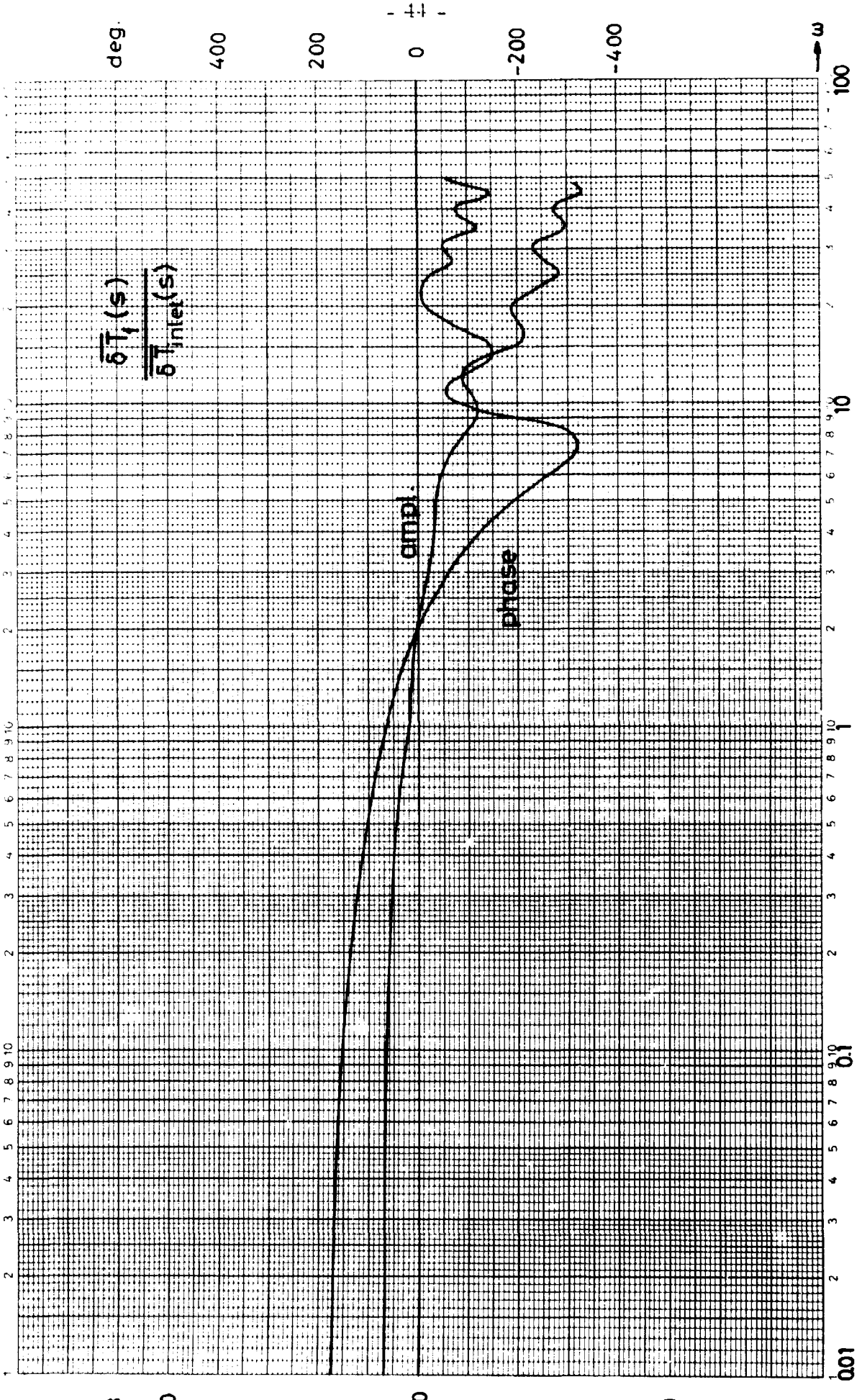
deg.
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
ampl.
phase
$\frac{\overline{\delta T}_1(s)}{\overline{\delta T}_{inlet}(s)}$

Fig.12

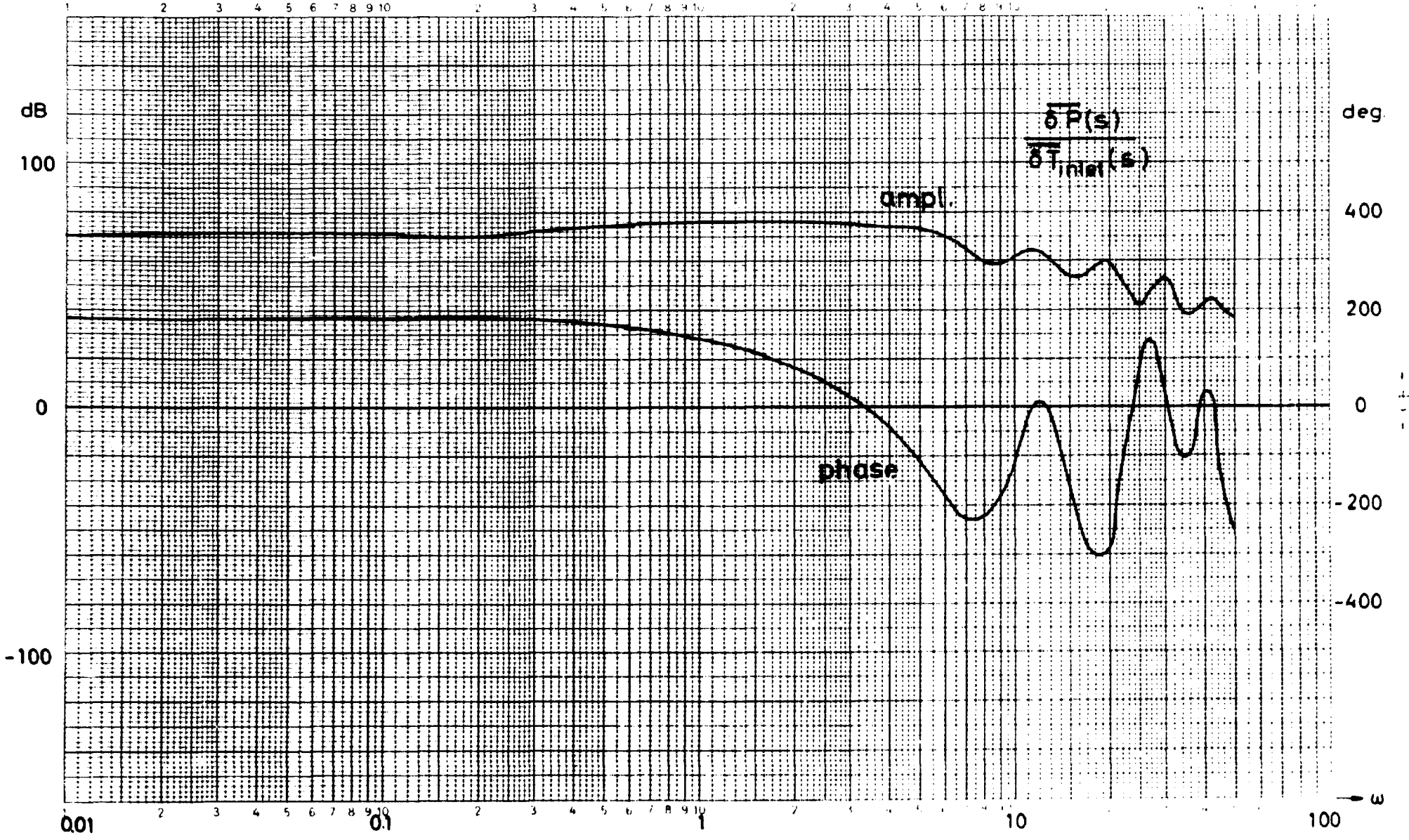
dB
100
0
-100
deg
400
200
0
-200
-400
ω
0.01
0.1
1
10
100
ampl.
phase
$\frac{\overline{\delta P(s)}}{\overline{\delta T_{inlet}(s)}}$

Fig.13

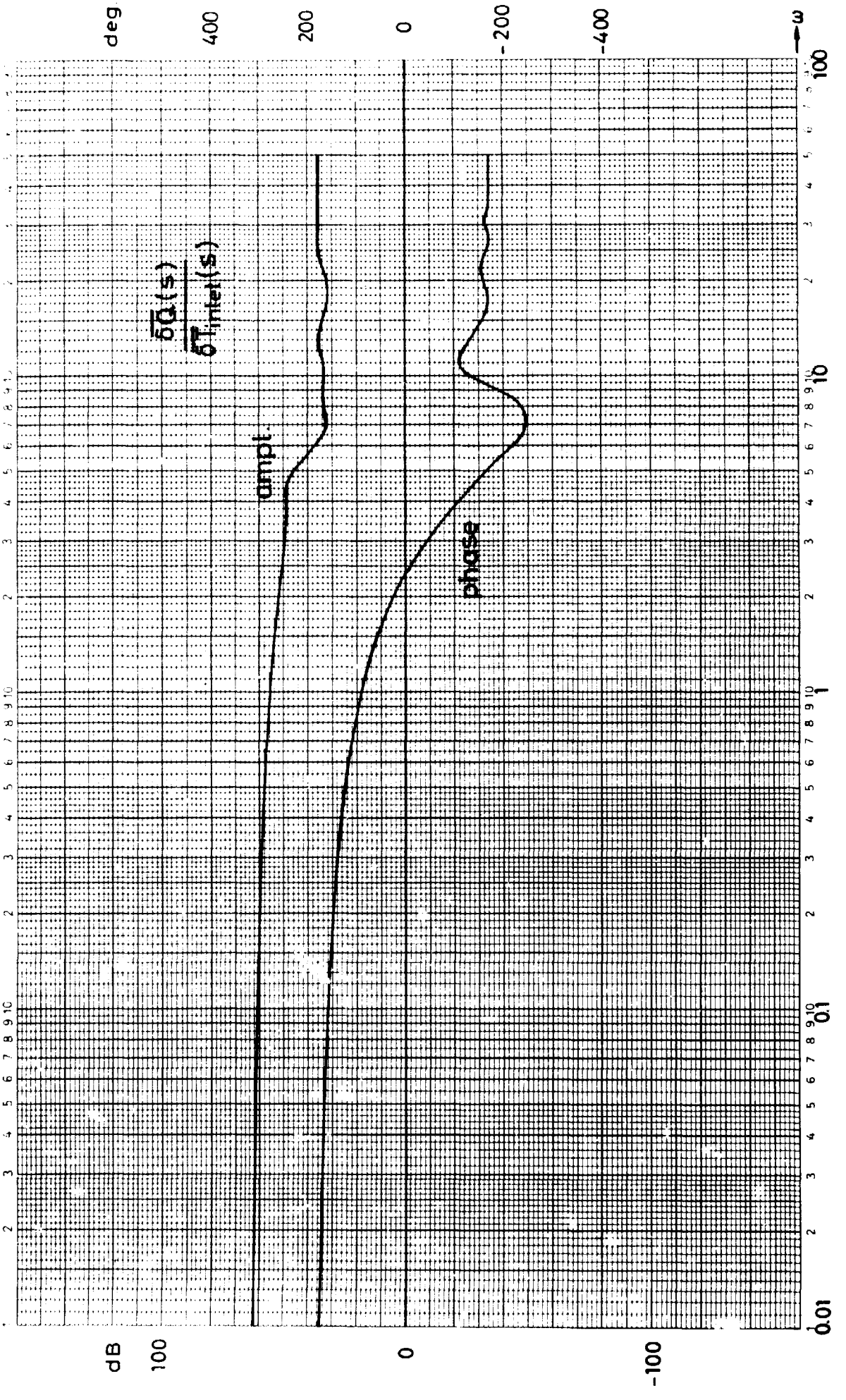
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
ampl.
phase
$\frac{\delta\bar{Q}(s)}{\delta\bar{T}_{inlet}(s)}$

Fig. 14

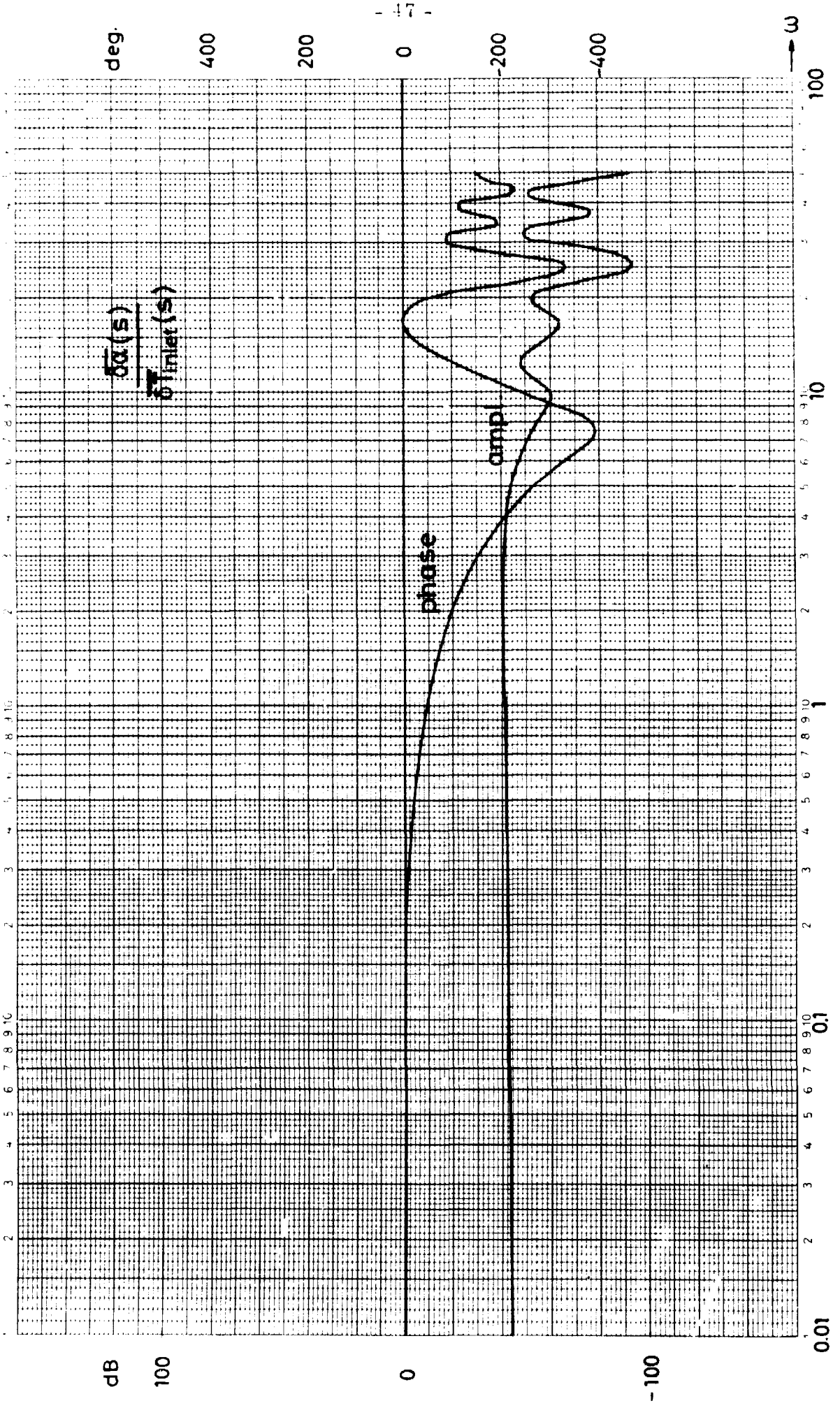
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
$\frac{\delta\alpha(s)}{\delta T_{Inlet}(s)}$
ampl
phase

Fig.15

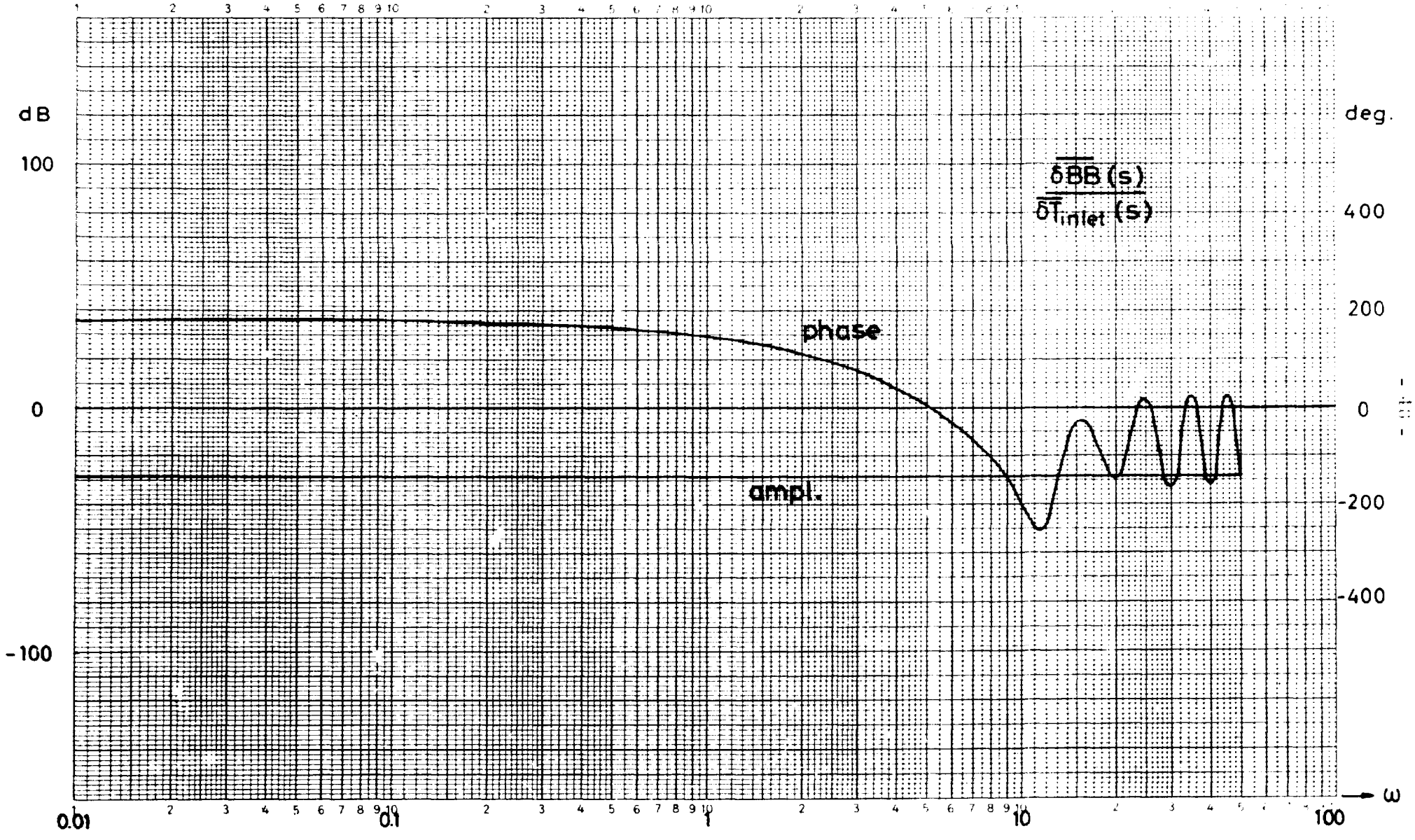
dB
100
0
-100
deg.
400
200
0
-200
-400
$\frac{\overline{\delta BB}(s)}{\overline{\delta T_{inlet}}(s)}$
phase
ampl.
ω
0.01
0.1
1
10
100

Fig.16

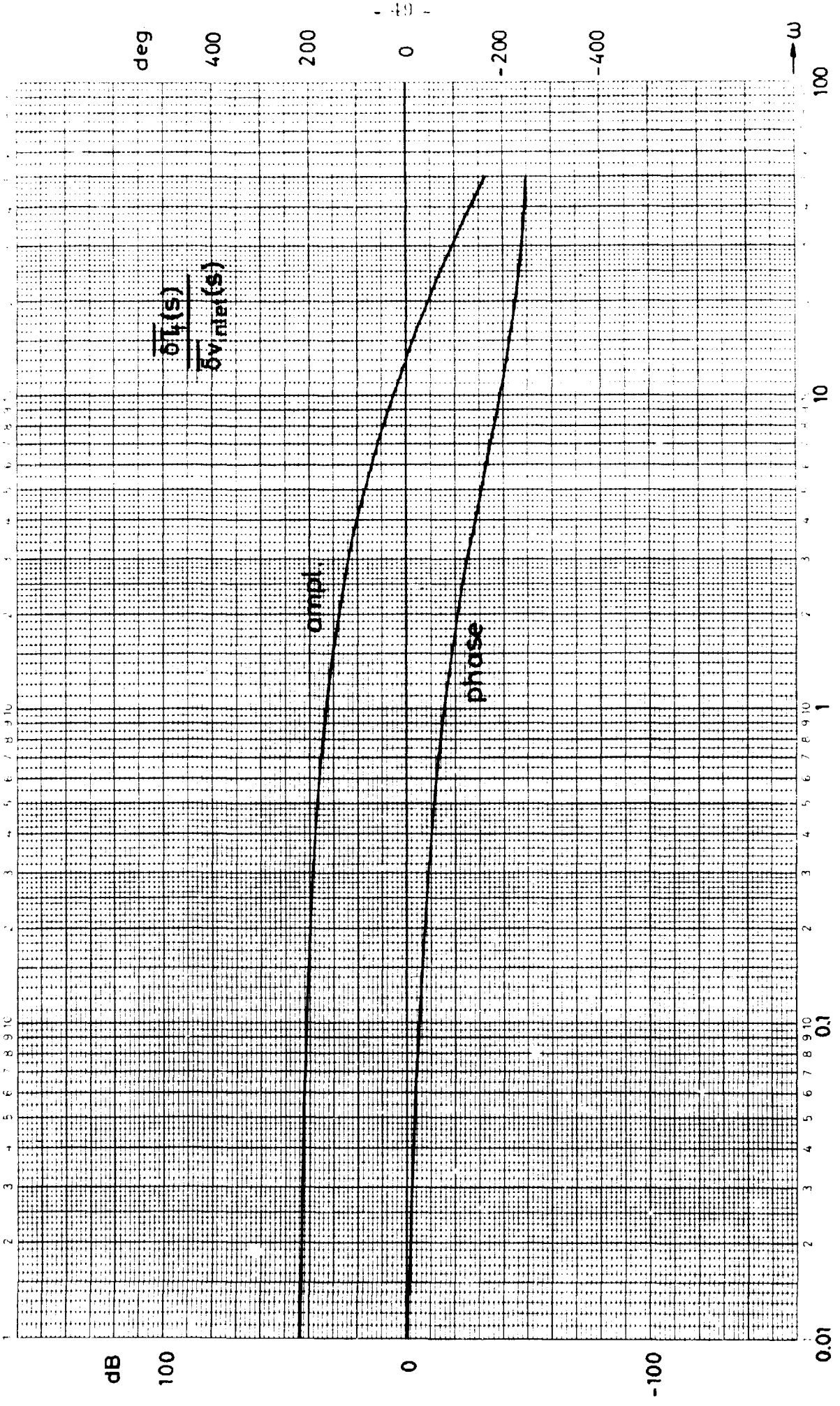
dB
100
0
-100
0.01
0.1
1
10
100
ω
deg
400
200
0
-200
-400
ampl.
phase
$\frac{\overline{\delta T_4(s)}}{\overline{\delta v_{inlet}(s)}}$

Fig.17

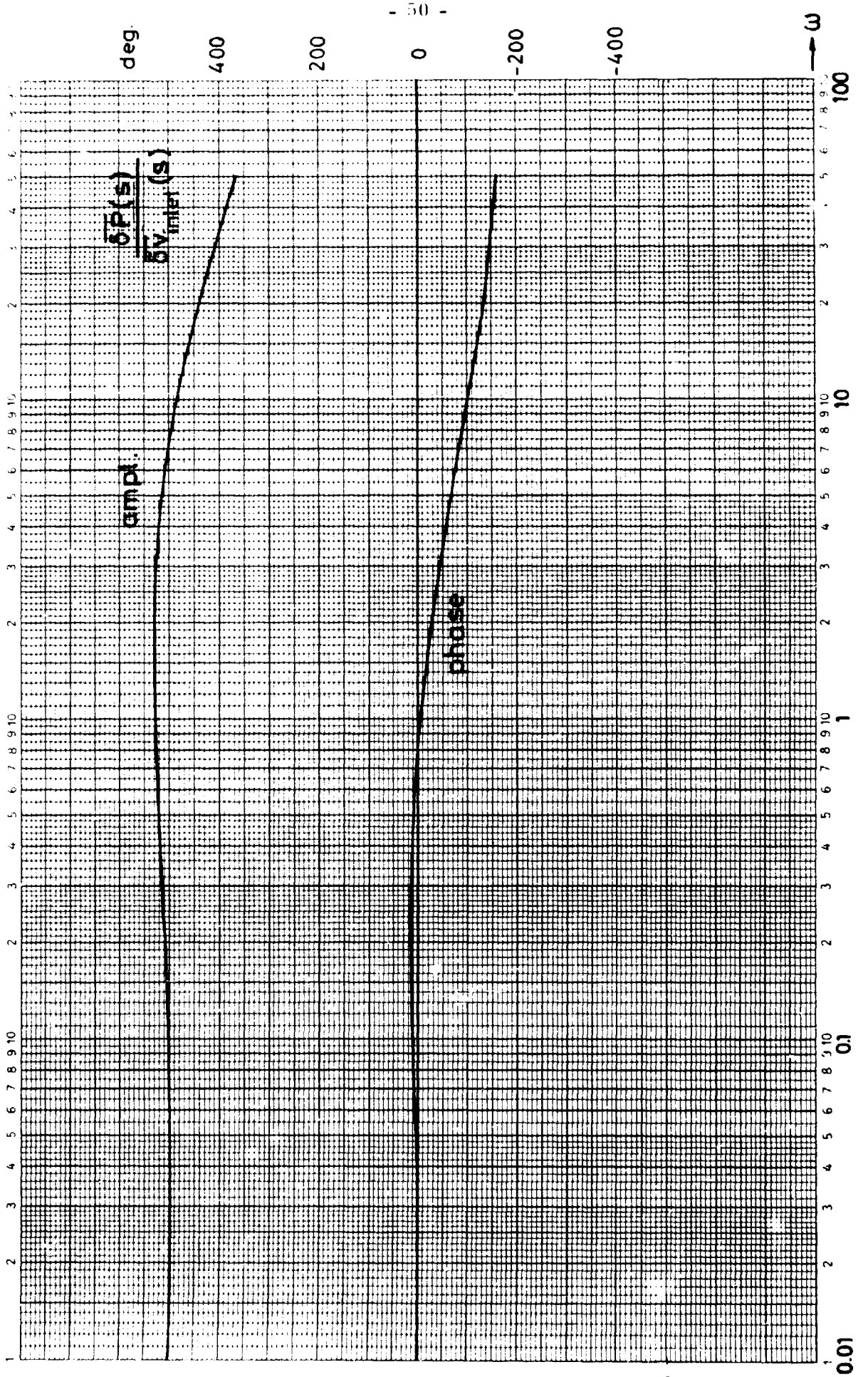
deg
400
200
0
-200
-400
ω
100
10
1
0.1
0.01
dB
100
0
-100
ampl.
phase
$\frac{\delta \bar{P}_c(s)}{\delta \bar{V}_{inlet}(s)}$

Fig.18

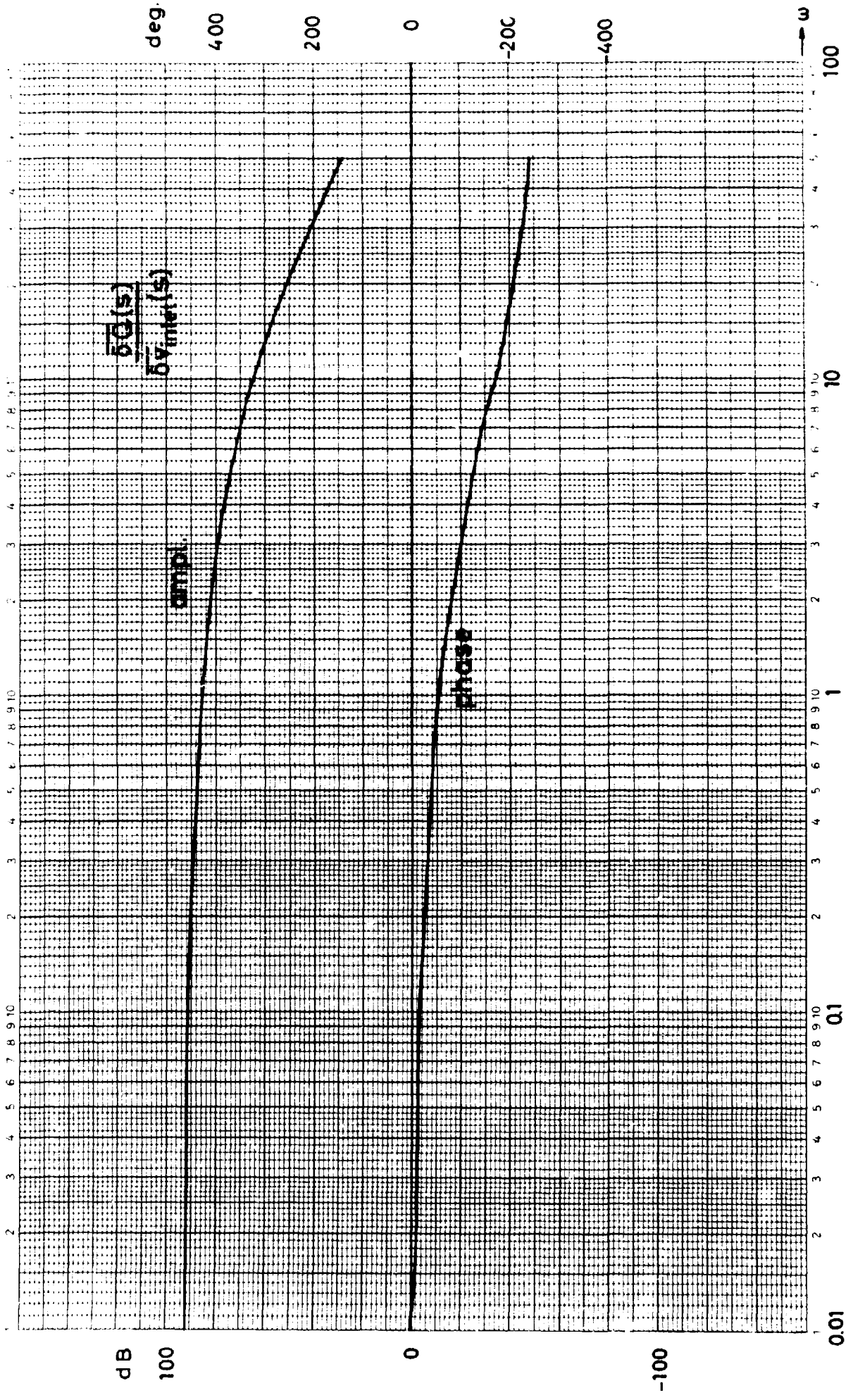
dB
100
0
-100
deg.
400
200
0
-200
-400
ω
0.01
0.1
1
10
100
ampl.
phase
$\frac{\overline{\delta Q}(s)}{\overline{\delta v_{inlet}}(s)}$

Fig.19

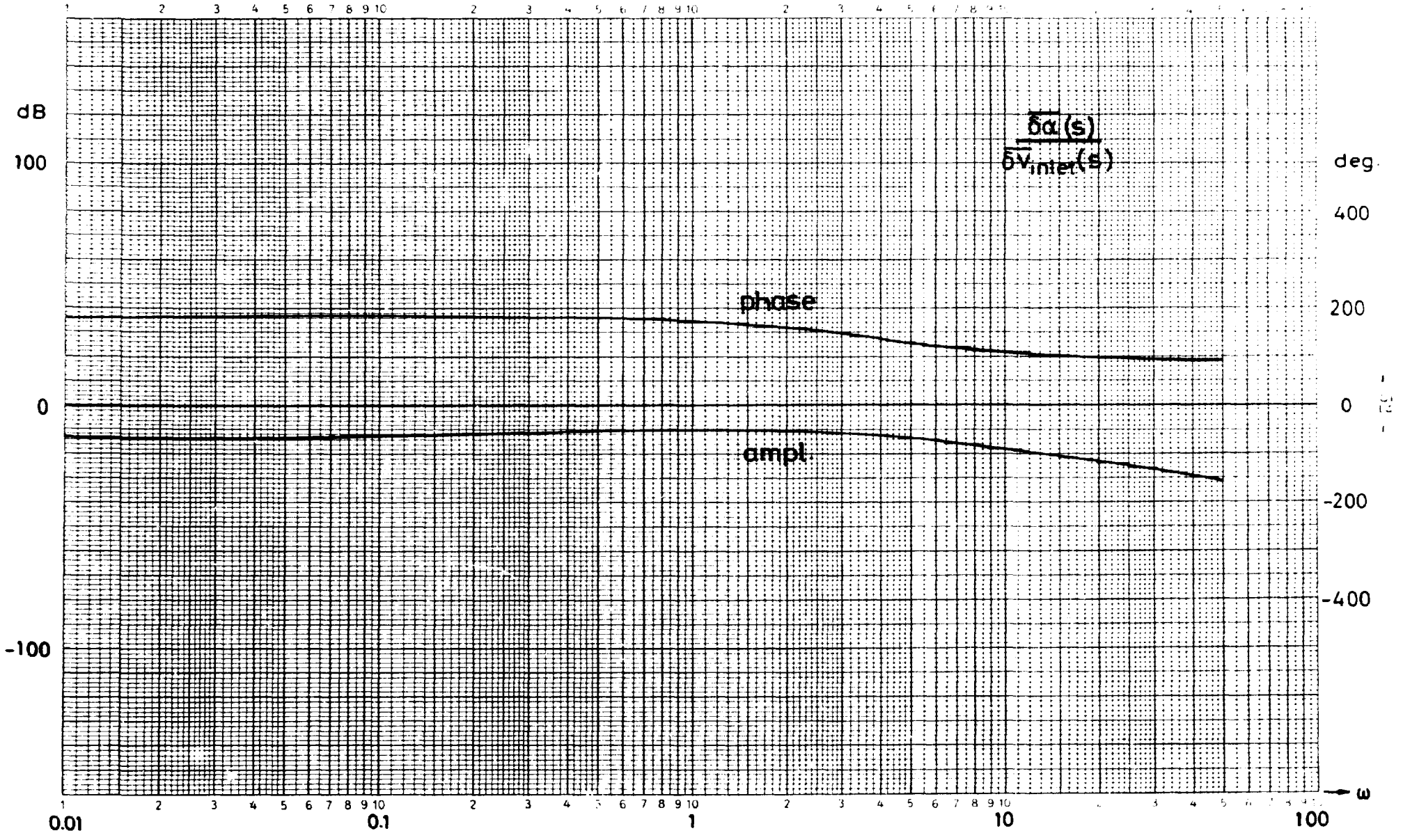
dB
100
0
-100
deg.
400
200
0
-200
-400
phase
ampl.
$\frac{\delta\alpha(s)}{\delta V_{inlet}(s)}$
ω
0.01
0.1
1
10
100

Fig. 20

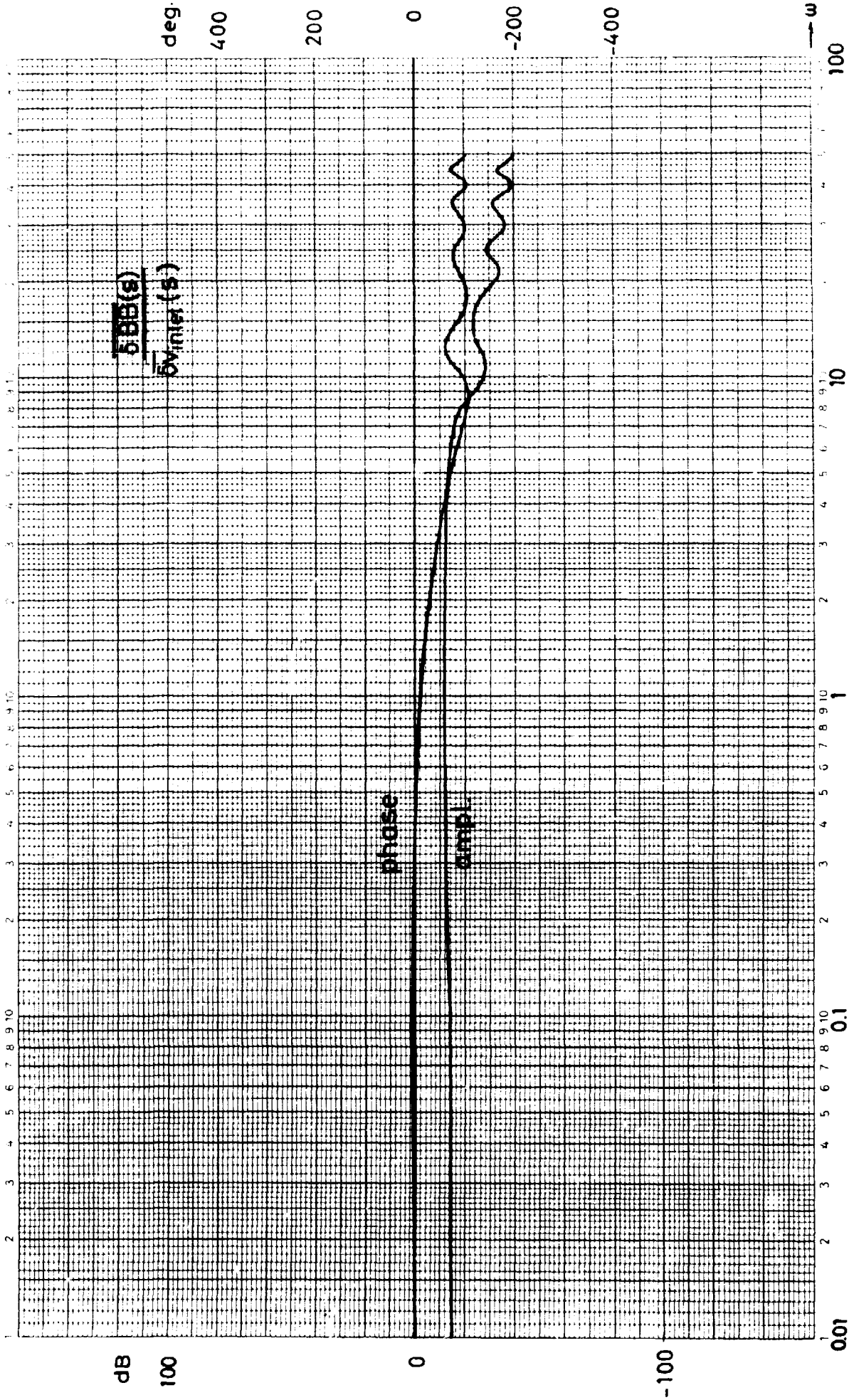
dB
100
0
-100
deg.
400
200
0
-200
-400
ω
0.01
0.1
1
10
100
phase
ampl.
δBB(s)
δv_inlet(s)

Fig. 21

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